

NASA CONTRACTOR REPORT 191422

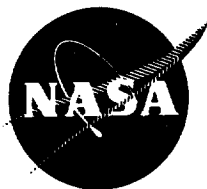
1N-24
158 548
P.134

**APPROXIMATING THE STRESS FIELD WITHIN
THE UNIT CELL OF A FABRIC REINFORCED
COMPOSITE USING REPLACEMENT ELEMENTS**

R. L. Foye

**Lockheed Engineering and Sciences Company
Hampton, VA 23666**

**Contract NAS1-19000
February 1993**



**National Aeronautics and
Space Administration**

**Langley Research Center
Hampton, Virginia 23681-0001**

(NASA-CR-191422) APPROXIMATING THE
STRESS FIELD WITHIN THE UNIT CELL
OF A FABRIC REINFORCED COMPOSITE
USING REPLACEMENT ELEMENTS Final
Report (Lockheed Engineering and
Sciences Corp.) 134 p

N93-23877

Unclass

G3/24 0158548

Table of Contents

	Page
List of Symbols	ii
Abstract	v
I Introduction	1
II One Dimensional Analysis	3
Sample Problem #1	3
III Two Dimensional Analysis	7
Sample Problem #2	8
Sample Problem #3	12
Sample Problem #4	13
IV Three Dimensional Analysis	16
Sample Problem #5	17
Sample Problem #6	18
Sample Problem #7	19
V Conclusions	25
References	26
Tables	27
Figures	35
Appendix A	A1
Appendix B	B1
Appendix C	C1
Appendix D	D1
Appendix E	E1
Appendix F	F1

List of Symbols

A	Area
B,[B]	Strain/displacement matrix
B.C.	Boundary condition
C,C _i	Cosine of angle in degress
D,[D]	Stress/strain coefficient matrix
\bar{D}_{ij}	Elements of $[\bar{D}]$
dx,dv	Increment of length or volume
E,E _i	Young's modulus
E _p	Epoxy
F,F _i	Nodal force component
F.E.	Finite Element
G,G _i ,G _{ij}	Shear Modulus
G _L	Glass
G _R	Graphite
i,j,k,m,n	Subscripts, superscripts, indices
[K], [k]	Stiffness matrix
L,l	Length
MP	Material property array or matrix
NM	Number of materials
NBX,NBY,NBZ	Number of elements along a coordinate axis
P	Load
[Q]	Stress/strain coefficient matrix
Q _i ,Q _{mn} ⁱ	Elements of [Q]
R	Radius
S,S _i	Sine of angle in degrees
S _{YM}	Symmetric

$[T]$	Stress or strain transformation matrix
$[\bar{T}]$	Inverse of $[T]$
u, v, w	Displacement components
v_l, v_f	Percent length or volume
x, y, z	Global coordinate axes
$\bar{x}, \bar{y}, \bar{z}$	Local coordinate axes
XYZ	Three dimensional woven reinforcement and composite
1,2,3	Principal coordinate of a material
2-D	Two-dimensional
3-D	Three-dimensional
γ, γ_{ij}	Shear strain component
δ	Deflection
ϵ, ϵ_i	Normal strain component
$\eta_{i,jk}$	Coefficeint of mutual influence of first kind
$\eta_{ij,k}$	Coefficient of mutual influence of second kind
σ, σ_i	Normal stress component
τ, τ_i, τ_{ij}	Shear stress component
ϕ, ϕ_i	Spherical angle specifying fiber direction
ψ, ψ_i	Spherical angle specifying normal to interfacial plane

Abstract

This report concerns the prediction of the elastic moduli and the internal stresses within the unit cell of a fabric reinforced composite. In the proposed analysis no restrictions or assumptions are necessary concerning yarn or tow cross-sectional shapes or paths through the unit cell but the unit cell itself must be a right hexagonal parallelepiped. All the unit cell dimensions are assumed to be small with respect to the thickness of the composite structure that it models..

The finite element analysis of a unit cell is usually complicated by the mesh generation problems and the non-standard, adjacent-cell, boundary conditions. This analysis avoids these problems through the use of preprogrammed boundary conditions and replacement materials (or elements). With replacement elements it is not necessary to match all the constituent material interfaces with finite element boundaries. Simple brick-shaped elements can be used to model the unit cell structure. The analysis predicts the elastic constants and the average stresses within each constituent material of each brick element. The application and results of this analysis are demonstrated through several example problems which include a number of composite microstructures.

I. Introduction

A unit cell of fabric reinforced composite is any small, closed, polygonal volume of inhomogeneous material (often brick shaped) which, when reproduced and similarly aligned, can be stacked, (side by side, top to bottom, and end to end) and joined together (as in solid brick construction) to approximate a variety of simple structural components whose minimum external dimensions are much larger than any unit cell dimension. Furthermore, it is desired that the thermo-mechanical response of the component and the unit cell assembly be similar. A variety of different unit cells and analyses have successfully predicted fabric reinforced composite moduli (Ref. 1) and average thermal properties (Ref. 2) but the resolution of the detailed internal stress distribution within a unit cell has been more difficult.

The ability to resolve the stresses within the unit cell of a fabric reinforced composite has at least three areas of applicability. Problems of crack growth within the microstructure are the most challenging of the three. The capacity to resolve the stress details must be very high in this application. Another level of usefulness is the prediction of the initiation and propagation of yielding or plastic flow (usually in the matrix phase) within the microstructure. This still requires a detailed knowledge of the internal stresses but it is not necessary to superimpose crack induced stresses on top of an already complicated stress field. A third, and much less demanding, level of usefulness is in material ranking and trade off studies. This level of engineering rates the likelihood of different fabric microstructures to perform satisfactorily in specific applications. Here the performance criteria can be quite simple and the demand for stress accuracy and detail can be significantly less than in the two prior applications. The large number of material and microstructural parameters available to the designer (or selector) of a fabric reinforced composite, coupled with the expense of experimentally characterizing these materials, makes initial screening by mechanical analysis more attractive. It was this application that was of most concern in the development of this analysis method. Numerical accuracy was clearly sacrificed to reduce modeling complexity in a manner consistent with material screening and comparison study requirements.

The three-dimensional stresses within a unit cell of a fabric reinforced composite can be predicted by the application of a general purpose finite element code. However, the associated boundary conditions on the unit cell surface and the mesh generation problems can be difficult. The program described in this report avoids these difficulties through the use of preprogrammed boundary conditions and replacement elements. With replacement elements it is not necessary to match all the internal material interfaces with finite element boundaries. Thus, simple, uniform, parallelepiped elements can be applied to a unit cell structure whose boundaries are themselves a

parallelepiped. Most of the common reinforcing microgeometries can be modeled with this shape of unit cell. The analysis predicts both the stresses (and strains) within each homogeneous element, and the average stress (and strain) within each dissimilar material contained in each replacement element. Conventional yield or failure criteria can then be applied to each material in each element, as in conventional stress analysis.

The proposed analysis places no restrictions on fabric microgeometry within the unit cell except that the fibers all be continuous, the fiber packing within any tow remain relatively constant, and the microgeometry be deterministic.

The key to the usefulness of this analysis is the performance of the replacement elements. This performance will be investigated for several sample problems of increasing complexity. These sample problems also help to explain the analysis and its application. The discussion begins with a simple one-dimensional tension bar problem. At this level the analysis seems almost trivial. The extension to two and three-dimensional problems is not trivial. In some of the sample problems the exact solution for the internal stresses is known. The plain weave unit cell is the most complex of the sample problems. For comparison, another numerical solution to this problem is available from an earlier study.

The two and three-dimensional problems require a computer analysis. The final version of this numerical analysis, as it evolved from a sequence of programs directed at each sample problem, is a Fortran program written for the Sun Spark station 1⁺. All of the equations and derivations for the two and three-dimensional analyses, along with the program listing and input/output descriptions, appear in the Appendices.

This analysis method and the related Fortran program, REPLACE, are considered to be an update of the earlier analysis program, FABNEW, which was developed about four years ago (Ref. 1). However, the earlier program has a thermal expansion prediction capability that could not be incorporated into REPLACE due to time and schedule limitations.

II . One Dimensional Analysis

In this section, the application and characteristics of replacement finite elements will be introduced at the simplest level, namely one-dimensional elastic analysis. Through the example of a tension bar, the convergence of various finite element models for the elastic deformations will be investigated and compared to the known solution. The proposed replacement element analysis is also capable of predicting average stresses in each constituent material within each element. The accuracy of these stress predictions are considered. There is no direct computational advantage to the use of replacement elements to model such a simple problem but it is instructive to initially consider the use of these elements at this elementary level.

Sample Problem #1

Consider the tension bar of Figure 1 in which the left hand half is made from a homogeneous isotropic material with modulus E and cross-sectional area A. The other half has the same cross-sectional area but the material is five times stiffer. From elementary considerations the total elongation of the bar (δ) is given by the sum of the elongations of the two halves.

$$\delta = \frac{P}{AE} \left(\frac{L}{2} \right) + \frac{P}{5AE} \left(\frac{L}{2} \right) = \frac{3}{5} \frac{PL}{AE}$$

where P is the axial load and L the total length of the bar. The axial stress (σ) and strain (ϵ) in each material are given by

$$\begin{aligned}\sigma_L &= \sigma_R = \frac{P}{A} \\ \epsilon_L &= 5\epsilon_R = \frac{P}{AE}\end{aligned}$$

where subscripts R, L designate right and left.

The same results could also have been obtained using finite element analysis as long as one of the finite element nodes coincided with the material discontinuity. In that case all of the elements would be homogeneous and their stiffness matrices precise, as long as the assumed displacement mode shapes included a constant and a linear term. The stiffness matrix [k] relevant to the axial forces and displacements at the end points of the bar is given by

$$[k] = \frac{5AE}{3L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

If the material discontinuity does not coincide with a node point then one element will be inhomogeneous, as shown in Figure 1, and the finite element solution will be an approximate one, as long as the assumed displacements are simple polynomials. The accuracy and convergence depends on the choice of mode shapes. For example, consider a linearly varying displacement within each element and an internal node placement at the 1/3 and 2/3 points along the bar length, as shown in Figure 1. Each subsequent refinement of the finite element grid divides each prior element into three equal segments. The middle element of the model will always be inhomogeneous as the element size decreases. The stiffness matrices for the homogeneous elements are given by

$$[k] = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

where l is element length.

The stiffness matrix for the single inhomogeneous element could be obtained from the general energy formula (Ref. 3)

$$[k] = \iiint_{VOL} B^T D B dv = A \int_0^L B^T D B dx \quad \text{————— (1)}$$

where B is the strain/displacement matrix and dx (dv) is an increment of length (volume) along the bar. D is the local material stress/strain relation. Superscript T designates transpose of a matrix. The resulting inhomogeneous bar stiffness matrix is given by

$$[k] = \frac{3AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

Figure 2 is a plot of the error in the bar elongation prediction as element size diminishes. The predicted end displacement approaches the known solution monotonically as the influence of the single inhomogeneous element error diminishes with element length. However, the error in the average strain of the center element persists at a high level (80%). This error can be reduced by resorting to higher order elements; but there is no accepted method for obtaining either the average or the detailed strains or stresses in the constituent materials within the inhomogeneous element.

Now consider a different approach to the same problem. Instead of applying the energy formula for the stiffness matrix, replace the inhomogeneous material with a fictitious homogeneous material that matches the axial response of the inhomogeneous materials. The center element is obviously a case of "stiffness in series", for which an equivalent modulus (\bar{E}) can be obtained from the rule of mixtures for stiffnesses in series (Ref. 4):

$$\bar{E} = \frac{E_L E_R}{v_L E_R + v_R E_L}$$

where v_i stands for fractional length of the i th segment of the element. For the particular example at hand where $E = E_L = E_R/5$ and $v_L = v_R = 0.5$

$$\bar{E} = \frac{5E}{3}$$

If this equivalent modulus is used for the center element the exact solution results. What is better, the "stiffness in series" model can be used to compute the average and local stresses and strains in the various materials of the inhomogeneous element from the nodal displacement solution. In particular, from Figure 1,

$$\sigma_L = \sigma_R = \frac{P}{A} \quad \text{and} \quad \epsilon_L = 5\epsilon_R = \frac{P}{AE}$$

which is the correct result.

This process of substituting equivalent homogeneous elements in place of inhomogeneous ones is termed the "replacement element" method.

Of course, if the "element in series" results were known, a priori, there would have been no need to resort to a finite element solution. However, in more complicated two and three-dimensional problems, knowing the local solutions for series and parallel stiffness models is not equivalent to solving a global problem that involves their use in place of inhomogeneous elements. For example, if the tension bar of Figure 1 were part of a redundant truss problem a truss analysis would still be required.

The error inherent in the use of the general energy formula, in combination with a low order displacement mode shape assumption, arises from the formula's inability to distinguish between series and parallel stiffnesses. For one-dimensional problems, with linear displacement assumptions, the energy formula presumes a "stiffnesses in parallel" situation, whether that is the case or not. The introduction of higher order displacement modes permit the general energy formula to make the necessary distinction. However, for polynomial mode shapes and

discontinuous material properties, the convergence rate improvements are slow and detailed stress and strain determination problems remain.

III. Two Dimensional Analysis

This section applies the concept of substituting replacement homogeneous elements in place of inhomogeneous ones at the generalized plane strain level of two-dimensional analysis. As in the one-dimensional case, the approach is first illustrated through a specific example for which the exact solution is easily obtained.

There are several different notions that should be introduced in the transition from one to two dimensions. The first is the unit cell concept. Much of the earlier work (Ref. 5) on the resolution of detailed stress fields in unidirectional materials (and laminates built up from unidirectional plies) used this type of idealization to make a large random microgeometry amenable to deterministic analysis. The unit cell approach looks for the simplest essential volume of composite microstructure from an analysis viewpoint. In two-dimensional analysis this selection is usually easy. Ref. 5 considered some convenient unit cells for square and hexagonally packed unidirectional composites. Each of the three sample problems in this section will begin by defining one or more unit cells for subsequent analysis. There are an infinite number of possible unit cells for a typical composite microstructure so the final choice is often somewhat personalized. The smallest unit cell is not always the most convenient one if the boundaries are non-rectangular.

Another basic difference between one and two-dimensional problems is the mathematical nature of the replacement element idealization. In one dimension the material interfaces are discrete points. Continuity of normal stress and the geometric relationship between average element normal strain and average constituent normal strains are the only relevant concerns. In two-dimensional analysis the constituent material interfaces are assumed to be linear (or planar) with several local stress and strain components of concern.

The physical nature of the replacement element process also changes from series and parallel bar or rod models to parallel plate models. The use of the general energy formula from Eq. 1 (as applied to a two-dimensional finite element) in combination with low order displacement mode shapes lead to the tacit assumption that each constituent material is arranged in a stacking of thin plates parallel to the plane of the analysis. The dissimilar material plates have their thicknesses in proportion to their respective volume fractions in the element. In reality, the constituent material interfaces are not parallel to the analysis plane but normal to it. The replacement element process corrects this inconsistency by rotating the same stacking of plates 90° about the material interface such that the final set of interfacial planes, between the parallel plates, preserves the original angle of the interface in the plane of the analysis. This procedure can only be applied to two constituent materials at a time whose interface is a single straight line in the

plane of the analysis. Thus, while the energy formula preserves only the constituent material volume fraction, the replacement element process preserves both the constituent volume fraction and the direction of the interface. Only the order or sequence of constituent material positioning across an interface is lost in the idealization. This process is best understood by considering the specific examples that follow.

Sample Problem #2

Figure 3 shows a laminated composite consisting of parallel bonded sheets of two different homogeneous isotropic materials. On a gross scale this assemblage of plates may be considered to be a composite material with a plane of isotropy parallel to the material interfaces. The principal axes of the composite are any pair of axes in the plane of isotropy with a third axis normal to that plane. In the principal axes, or natural coordinates of the composite, the elastic constants can be established from the application of elementary mechanics principals to the unit cell structure. Also, the same elementary model can be used to obtain the equations for the internal stresses in each constituent material corresponding to any remotely applied state of uniform composite stress or strain. The elastic constants and the detailed stresses and strains can then be transformed into any global reference system: in particular, the one shown in Figure 3 where one of the natural coordinates correspond to the z-axis of the global reference system.

The isotropic properties of the two sets of parallel plates can be chosen to match the properties of aluminum and epoxy from Table 1. The volume fractions of both constituents are 0.5. From elementary mechanics considerations the elastic constants of the composite, in the principal axes, can be obtained as follows. Consider the unit cell of Fig. 3 in the 1,2,3 coordinate system. From equilibrium and resolution of forces the average composite stresses ($\bar{\sigma}_i, \bar{\tau}_{ij}$) are related to the constituent stresses (σ_i^k, τ_{ij}^k) by

$$\begin{aligned}\bar{\sigma}_1 &= \sigma_1^{Al} V_{Al} + \sigma_1^{Ep} V_{Ep} \\ \bar{\sigma}_2 &= \sigma_2^{Al} = \sigma_2^{Ep} \\ \bar{\sigma}_3 &= \sigma_3^{Al} V_{Al} + \sigma_3^{Ep} V_{Ep} \\ \bar{\tau}_{12} &= \tau_{12}^{Al} = \tau_{12}^{Ep}\end{aligned}$$

where v_i designates volume fraction of the i th constituent and Ep and AL designate epoxy and aluminum respectively. The corresponding strains ($\bar{\epsilon}_i, \bar{\gamma}_{ij}, \epsilon_i^k, \gamma_{ij}^k$) are related by geometry and compatibility as follows

$$\begin{aligned}\bar{\epsilon}_1 &= \epsilon_1^{Al} = \epsilon_1^{Ep} \\ \bar{\epsilon}_2 &= \epsilon_2^{Al} V_{Al} + \epsilon_2^{Ep} V_{Ep} \\ \bar{\epsilon}_3 &= \epsilon_3^{Al} = \epsilon_3^{Ep} \\ \bar{\gamma}_{12} &= \gamma_{12}^{Al} V_{Al} + \gamma_{12}^{Ep} V_{Ep} .\end{aligned}$$

These 12 equations plus the individual stress/strain laws for the two constituent materials form a system of 20 equations that can be solved for the composite stress/strain relation and the individual constituent stresses and strains corresponding to any applied composite stresses or strains (see Appendix A). From the composite stress/strain relations the composite elastic constants are

$$\begin{aligned}E_1 &= E_3 = 5.25 \times 10^6 \text{ psi} \\ E_2 &= 1.39 \times 10^6 \text{ psi} \\ G_{12} &= 0.354 \times 10^6 \text{ psi} \\ \nu_{12} &= \nu_{32} = 0.325 \\ \nu_{13} &= 0.255\end{aligned}$$

From these principal values the engineering constants in another coordinate system, obtained by a rotation about the 3-axis of Figure 3, can be calculated from the appropriate 2-D transformation equations (given in Appendix B). In particular, for a rotation of 45° about the 3-axis of Figure 3 the elastic constant are

$$\begin{aligned}E_x &= E_y = 1.11 \times 10^6 \text{ psi} \\ E_z &= 5.25 \times 10^6 \text{ psi} \\ G_{xy} &= 0.968 \times 10^6 \text{ psi} \\ \nu_{xy} &= 0.566 \\ \nu_{xz} &= \nu_{yz} = 0.067 \\ \eta_{xy,x} &= \eta_{xy,y} = -0.296 \\ \eta_{xy,z} &= 0.034\end{aligned}$$

For an average composite tensile stress of one psi in the x-direction (with all the other average stress components equal to zero) the stresses in the constituent materials are given by

	Aluminum	Epoxy
σ_x	1.103	0.897
σ_y	0.103	-0.103
σ_z	-0.251	0.251
τ_{xy}	0.103	-0.103

These stress and moduli predictions from elementary analysis are exact because they can be shown to satisfy all the local and global conditions of equilibrium and compatibility.

As in the one-dimensional example, these results can also be obtained by conventional finite element analysis using various types of elements and grids. The unit cell can be analyzed in the principal coordinates of the material, as shown in Figure 4, using rectangular or constant strain triangular elements without violating element material homogeneity. The applied unit stress in the x-direction can be resolved into its components in the 1,2,3 coordinates of Figure 4 by either a Mohr's circle or the use of the stress transformation equations of Appendix B. The resulting composite moduli and constituent stress predictions can then be transformed back into the global x,y,z coordinate system. These results agree precisely with the results of the elementary analysis.

Alternatively, using the unit cell and grid of Figure 5A, with constant strain triangular elements, the exact results can be obtained from homogeneous elements without the necessity of transforming the input and output from one coordinate system to another.

It is interesting to also consider the application of inhomogeneous finite elements to the analysis of the same unit cell. Figure 5B shows this unit cell of the composite and one possible subdivision of the unit cell into rectangular elements. Some of the elements are homogeneous and some inhomogeneous. Using 4-node, isoparametric, brick elements (Ref 3); generalized plane strain analysis; the 25-node finite element grid shown in Figure 5B; and the general energy formula (Eq. 1) for the stiffness matrix of the inhomogeneous elements, the analysis overestimates the x and y moduli by almost 100%. Refinement of the grid leads to the moduli

estimates of Figure 6. The convergence is slow. Furthermore, there is no effective method of obtaining constituent material stresses within the inhomogeneous elements.

Now consider replacing the inhomogeneous elements in this example problem with replacement elements. To make this substitution in two dimensions first consider a subelement of the inhomogeneous material, shown in Figure 7. The sides of this subelement are either parallel or normal to the material boundary plane. The volume fractions of the two materials are the same in the subelement as in the element that contains it. Assume that the replacement homogeneous material for the subelement and the whole element are the same. The derivation of Appendix A then can be applied to establish both the replacement homogeneous material moduli and the average constituent material stresses, once the average element strains are established. The physical nature of the homogeneous-inhomogeneous replacement process is now evident. The inhomogeneous element of Figure 7 is replaced by a homogeneous composite element consisting of parallel plates bonded together in the same volume fraction as the inhomogeneous element and having the same orientation of the material interfaces. With the 25-node finite element grid the substitution is of the nature shown in Figure 8. For simplicity let the rectangular element stiffness matrix be made up of the sum of two constant strain triangular elements. (There is no need for higher order elements in this example.) The same replacement material substitution is done for both of the constant strain triangles that make up the rectangular element. The stress predictions for the constituent materials in the rectangular element are the average values from the two triangles.

The results from the 25-node finite element analysis are not the same as the exact solution for either the moduli or the constituent stresses. The Young's modulus in the loading direction is 31% high as a result of the use of the replacement elements. This is a considerable improvement over the 100% error using the same finite element grid with the general energy formula for element stiffness. This error diminishes to less than 14% if the rectangular grid is changed from 4x4 to 8x8 as shown in Figure 9. Since the replacement element analysis also provides constituent stresses it is of interest to compare the stresses in the 4x4 replacement elements to the known results. The following table makes this comparison.

	Aluminum Phase		Epoxy Phase	
	Replacement Element	Exact Result	Replacement Element	Exact Result
$\sigma_x(\text{psi})$	0.906	1.103	0.677	0.898
$\sigma_y(\text{psi})$	0.168	0.103	-0.081	-0.103
$\sigma_z(\text{psi})$	-0.264	-0.251	0.189	0.251
$\tau_{xy}(\text{psi})$	0.155	0.103	-0.075	-0.103

The peak stresses from the replacement elements are about 20% lower than the exact values. Unfortunately, as in the one-dimensional case, these constituent stress errors do not diminish with grid refinement. These errors must be reduced by the use of improved elements. The stresses in the homogeneous elements away from the replacement elements do converge rapidly to the exact results with increasing grid refinement.

As was true in the tension bar example, the use of the general energy expression for the inhomogeneous element stiffness matrix, in combination with low order displacement mode shapes, favors an "elements in parallel" model of behavior rather than an "elements in series" model as is sometimes more appropriate. Figure 10 illustrates this tendency of an inhomogeneous plane stress element (by reference to a lattice or framework model). If the upper and lower halves of the element, as shown in Figure 10A, were made of dissimilar isotropic materials then good engineering judgment would dictate the lattice representation of Figure 10B, where lattice members that cross the material boundary are modeled as "elements in series" while those that do not cross the material boundary are simply homogeneous. The low order energy formula leads to a lattice structure of the type shown in Figure 10C. If there is not much difference between the stiffness of the constituent materials the two lattice models do not differ significantly. But if the constituents are very different, elastically, then the two models differ widely.

Sample Problem #3

This sample problem involves the determination of the extensional moduli and fiber/matrix stress concentrations for a unidirectional composite consisting of a square packed array of glass fibers in an epoxy matrix. These stiffnesses and stress concentrations are well established from several earlier micromechanics investigations. It will be shown that finite element analysis based on the substitution of orthotropic replacement elements for the inhomogeneous elements can yield approximately the same results for both moduli predictions and stress analysis even though the stresses within any constituent material in the unit cell model are not uniform.

The specific problem concerns a 50% fiber volume fraction of unidirectional E glass in an epoxy matrix. Figure 11 shows the square packed array of fiber cross-sections and a single unit cell of the composite. At most, only one quadrant of the unit cell needs to be analyzed due to structural and load symmetry. The constituent material properties are given in Table 1.

The 5 x 5 rectangular finite element grid of Figure 12 is superposed on the fiber/matrix geometry. The rectangular, generalized plane-strain, element stiffness matrices are formed from a pair of constant strain triangular elements, using the same replacement material properties in each

triangle of the rectangle. This leads to the material model of Figure 13 in which the plate thickness and spacing within each originally inhomogeneous element reflects the true constituent volume fractions and the approximate interfacial geometry (with the cylindrical interfacial surfaces replaced by flat planes).

Figure 14 contains contour plots of the stresses in the epoxy matrix due to a remote unit average tensile stress normal to the fiber principal axis. The stress distributions in the glass fibers are somewhat featureless. The stresses in the inhomogeneous elements were treated the same as the homogeneous element stresses in preparing the contour plots. For comparison, the same distribution of matrix stresses is also given in Figure 15 from Reference 5. The latter stresses were established using a conventional finite element analysis in which all the elements were homogeneous and isotropic. The stress distributions are essentially the same except for a slightly higher replacement element stress concentration at the fiber/matrix interface along a line of closest approach of adjacent fibers in the loading direction. This shows that the replacement scheme can give accurate stresses when the stresses and strains within the constituent materials are nonuniform. Furthermore, it is not necessary to resort to more refined grids in order to obtain comparable stress predictions.

The transverse Young's modulus prediction from the replacement element solution was 1.8 million psi. This also compares favorably with other published values for the same square-packed array of glass fibers. For example, Reference 4, lists a value of 1.7 million psi for a 50% fiber volume fraction glass/epoxy with similar constituent properties using conventional finite element analyses.

Sample Problem #4

This sample problem also represents a 2-D generalized plane-strain analysis in which the constituent material stresses are not uniform. However, the geometry of the reinforcement phase was chosen to resemble that of a wavy tow. This microgeometry has sometimes been chosen as representative of woven fiber unit cell microgeometries (References 4, 6). Figure 16 shows the idealized composite structure and a unit cell of that structure. The reinforcing phase consists of stacked layers of corrugated aluminum sheets separated by similar layers of epoxy. Perfect bonding is assumed between the two phases. The dimensions of the microstructure are given in Figure 17. The Young's modulus of the composite normal to the plane of Figure 16 can be predicted adequately by the rule of mixtures for elements in parallel, but the Young's moduli in the x or y- directions require a finite element analysis. This analysis will also consider the deformations and stresses in the unit cell as a result of some average strain in the x-direction, with

all other average strain components held to zero. The constituent material properties are given in Table 1. The volume fraction of the aluminum is 56%. From symmetry of the microstructure and loading only half of the unit cell needs to be analyzed.

In order to have a basis of comparison for the approximate analyses a detailed finite element analysis was performed on this microstructure using the NASTRAN code (Ref. 7) and the two grids shown in Figure 18. The coarse grid contains 20 elements. The refined grid has 676 elements. All the elements were homogeneous isotropic CHEXA2 or CWEDGE elements. Three independent unit strain cases were run in order to obtain average composite extensional properties and the corresponding stresses and deformations. The average strain case $\{\bar{\epsilon}_x = 1.0, \bar{\epsilon}_y = \bar{\epsilon}_z = \bar{\gamma}_{yz} = \bar{\gamma}_{xz} = \bar{\gamma}_{xy} = 0.0\}$ gave the required internal deformations and stresses. The strain cases $\{\bar{\epsilon}_z = 1.0, \bar{\epsilon}_x = \bar{\epsilon}_y = \bar{\gamma}_{yz} = \bar{\gamma}_{xz} = \bar{\gamma}_{xy} = 0.0\}$ and $\{\bar{\epsilon}_x = \bar{\epsilon}_y = \bar{\epsilon}_z = 1.0, \bar{\gamma}_{yz} = \bar{\gamma}_{xz} = \bar{\gamma}_{xy} = 0.0\}$ gave sufficient information to establish the extensional moduli. The last strain case was obtained by specifying that all average strains vanish and that both constituent materials have a unit coefficient of thermal expansion while the unit cell is subject to a one degree change in temperature. This was necessary to avoid the occurrence of constant displacement terms in the multi-point constraint equations at nodes that were located on surfaces of the unit cell where symmetry conditions did not apply (Ref. 7).

The generalized plane strain, extensional, elastic constants from the NASTRAN models are

	Coarse Grid	Fine Grid
Ex psi ($\times 10^6$)	3.55	3.09
Ey psi ($\times 10^6$)	1.48	1.43
Ez psi ($\times 10^6$)	5.81	5.83
ν_{zx}	0.25	0.26
ν_{zy}	0.32	0.32
ν_{yx}	0.20	0.24

The results from the fine grid are used as the basis of comparison for this example problem. Figures 19 and 20 contain plots of the unit cell surface normal deformations and internal stress components for the $\bar{\epsilon}_x \neq 0$ strain case. Many of the stress details of the fine grid are not evident in the coarse 20-element solution. Even with the refined grid it is not certain whether some of the peak stresses have been accurately quantified. The large amount of periodic local bending and shearing deformations in the reinforcing sheets are evident in the deformation plots. Large local bending stress gradients through the aluminum sheets are also evident in the stress plots. In brief,

the task of characterizing the response of this microstructure is a more complex problem than the previous example problem and represents a stiff test of the replacement element method.

First consider the inhomogeneous element modeling of this microstructure using the 4 x 4 grid of Figure 21 and the general energy formulation for the inhomogeneous element stiffness matrices. Using four-node, isoparametric, generalized plane strain elements, with the 16-element grid the extensional moduli estimates are

$$E_x = 4.90 \times 10^6 \text{ psi}$$

$$E_y = 3.12 \times 10^6 \text{ psi}$$

$$E_z = 5.84 \times 10^6 \text{ psi}$$

Except for E_z these estimates deviate significantly from the NASTRAN results. If the grid is refined from 4 x 4 to 8 x 8 as shown in Figure 21 the moduli values improve somewhat to

$$E_x = 3.95 \times 10^6 \text{ psi}$$

$$E_y = 2.32 \times 10^6 \text{ psi}$$

$$E_z = 5.88 \times 10^6 \text{ psi}$$

However, both the E_x and E_y moduli estimates remain beyond the desired bounds of engineering accuracy, and no internal stress data accompany these stiffness estimates. Both of these shortcomings can be remedied by the use of replacement elements.

From the NASTRAN stress results it is obvious that the 4 x 4 grid will not give sufficient detail to present any kind of comprehensive picture of the true stress distributions, no matter how accurate the replacement element results may be. Thus the 10 x 10 grid of Figure 21 is applied to the current problem with the same type of rectangular replacement element that was used in the previous sample problem. With this grid 18% of the elements are inhomogeneous. The resulting moduli estimates are

$$E_x = 3.21 \times 10^6 \text{ psi}$$

$$E_y = 1.62 \times 10^6 \text{ psi}$$

$$E_z = 5.89 \times 10^6 \text{ psi}$$

These values compare favorably to the base line NASTRAN results. Figure 22 presents the stress contours and unit cell surface normal deflections from the 10 x 10 replacement element analysis of the $\bar{\epsilon}_x \neq 0$ strain case. The approximations are remarkably consistent with, though slightly less detailed than, the fine grid NASTRAN results in Figure 20. The approximations are a major improvement in detail over the coarse NASTRAN stress results.

IV. Three Dimensional Analysis

The previous sections and example problems have hopefully established the credibility of the replacement element method at the one and two-dimensional analysis levels. This section extends the method to the 3-D level. Figure 23 shows a parallelepiped element containing two different constituent materials. The geometric configuration can be described by specifying the volume fraction of one (or both) constituent and the direction of a normal to the interfacial plane. The sequence in which the constituent materials appear, as an observer moves along the normal to the interfacial plane, is irrelevant to the replacement element method. Figure 23 illustrates the two spherical angles ψ_1, ψ_2 that specify the direction of the normal to the interfacial plane. These two direction angles also serve to locate a set of local coordinates $(\bar{x}, \bar{y}, \bar{z})$ parallel and normal to the interfacial plane. The \bar{y} and \bar{z} axes lie in the plane. \bar{x} is normal to it. The replacement element concept rearranges the two bulk constituents into a series of parallel plates with the plate surfaces paralleling the original interfacial plane. Normal and tangential shear stress continuity is preserved across the interface. Compatibility of normal strain in the \bar{y} and \bar{z} -directions and shear strain in the $\bar{y}\bar{z}$ plane (of Figure 23) is maintained across the interfaces.

Constituent material properties are treated more generally than in the 2-D case. Each constituent is assumed to be orthotropic with a plane of isotropy normal to the principal reinforcing direction. The principal reinforcing direction must be specified, by means of two spherical angles, ϕ_1 and ϕ_2 . These angles are referenced and measured in the same sense as the ψ_1 and ψ_2 angles of Figure 23 with the interfacial normal direction replaced by the grain (or fiber) direction of the constituent material. Usually the principal reinforcing direction will parallel the interfacial plane but this is not assumed in the analysis.

To form the stress/strain law for the replacement element a number of stress and strain transformations must be carried out. Each constituent material has its stress/strain relations initially specified in the natural coordinates of the material. These properties must be transformed into the x, y, z global coordinates first and then transformed into the $\bar{x}, \bar{y}, \bar{z}$ interfacial coordinates. The replacement analysis then yields the replacement material stress/strain law in the $\bar{x}, \bar{y}, \bar{z}$ coordinates. Finally, these properties are transformed back into the global x, y, z coordinates for use in constructing the element stiffness matrix. This sequence of transformations is retraced (after the finite element analysis of the unit cell yields node point deflections and average element strains in the global coordinates) in order to get constituent material stresses in the natural coordinates of the materials. Appendix C derives the replacement element stress/strain equations in the interfacial coordinates. Appendix D gives the transformation equations.

The 3-D stress and strain transformations are accomplished by a pair of essentially 2-D transformations. Each transformation accounts for each spherical angle of rotation that specifies either the direction of the normal to the interfacial plane or the fiber direction.

Sample Problem #5

As in the 1-D and 2-D case, the first 3-D sample problem is an elementary one for which a solution is available. However, in this case the known solution is not exact. The problem concerns the "3-D weave" or "XYZ" composite construction (see Figure 24) in which there are three orthogonal fiber directions (Ref 8). The fibers remain essentially straight. The volume fraction of fibers in each of the orthogonal directions usually vary to match the design requirements. The types of fibers may also vary with direction. Figure 24 shows one unit cell of the composite microstructure. Symmetry considerations reduce the essential part of the unit cell that must be analyzed to one eighth of the total unit cell volume. This reduced volume is shown in Figure 25. It has a 25% volume fraction of interstitial bulk matrix, a 25% volume fraction of unidirectional composite with fibers in the x-direction, a 37.5% volume fraction of composite in the y-direction and a 12.5% volume fraction of composite in the z-direction. The unidirectional material is taken to be graphite/epoxy with the properties listed in Table 2 under material A. The bulk epoxy properties are the same as in the prior sample problems. Using conventional, homogeneous, eight-node, isoparametric brick elements and the finite element grid of Figure 26A, the extensional composite elastic constants are

$$E_x = 5.49 \times 10^6 \text{ psi}$$

$$E_y = 7.55 \times 10^6 \text{ psi}$$

$$E_z = 3.43 \times 10^6 \text{ psi}$$

$$\nu_{yz} = 0.128$$

$$\nu_{xz} = 0.131$$

$$\nu_{xy} = 0.055$$

The average normal stress in the x-direction in each element as a result of an applied average tensile stress of 1000 psi in the global x-direction is given in Figure 27. The results are approximate because the stresses are not constant within each brick element.

The same problem can also be addressed using the replacement element approach. For example, if the finite element grid of Figure 26B were applied to the XYZ microgeometry there

would be three of the eight, equal -sized, brick elements that were inhomogeneous. Using the replacement element analysis of Appendix D the pairs of inhomogeneous material in each of these three elements can be resolved into three different replacement materials. Using one of these replacement materials in each of the inhomogeneous brick elements the finite element analysis can proceed as a homogeneous element analysis and the composite stiffnesses and average element strains obtained. The same replacement material model may then be used to obtain average constituent stresses and strains within each element. These stress predictions are given in Figure 28. A comparison of Figures 27 and 28 shows that the approximate results from the replacement element analysis are of considerable engineering value. The moduli predictions from the two models compare as follows:

	Homogeneous Elements	Replacement Elements
E_x	5.49×10^6 psi	5.46×10^6 psi
E_y	7.55×10^6 psi	7.55×10^6 psi
E_z	3.43×10^6 psi	3.44×10^6 psi
ν_{yz}	0.128	0.128
ν_{xz}	0.131	0.130
ν_{xy}	0.055	0.054

There are no stiffness discrepancies of any note between the models. The details of the input data are given in Appendix E where this sample problem is used to demonstrate the input data sequences for the interactive use of the replacement element computer code.

Sample Problem #6

The next 3-D sample problem represents a composite comprised of solid glass spheres in an epoxy matrix. The volume fraction of the glass reinforcing phase is 25%. The spheres are all the same size and are assumed to be packed in a cubic array as shown in Figure 29. The ratio of sphere diameter to the spacing distance between centers of adjacent spheres (in the direction of closest approach) is 0.684. The problem is the prediction of both the principal Young' modulus in the x-direction of Figure 29 and the peak normal matrix stress along the line of closest approach of adjacent spheres when the composite has an average remotely applied tensile loading of one psi in the x-direction, with all other average stress components equal to zero.

The problem has no known exact solution but a numerical solution could be obtained with any general purpose, 3-D, finite elements code based on the use of conventional, homogeneous,

isotropic elements. However, it is of current interest to obtain a solution using rectangular grids and replacement elements.

From symmetry considerations only one octant of a unit cell of the composite needs to be analyzed. Figure 30 divides this octant into cubic elements with the 4x4x4 subdivision shown. Each element is designated by an i,j,k combination of integers. The i integer indicates the element number along x-axis starting at the origin of Figure 30. j and k are the corresponding element counts along the y and z-axes respectively. The 1,1,1 element has one corner on the origin and the 4,4,4 element is the farthest one from the origin. Table 3 contains the spherical angles (ψ_1, ψ_2) that designate the direction of the outward pointing normals from the surface of the glass sphere in each element. The table also contains the element volume fractions that are glass and epoxy. This is all the input data that is necessary to compute the principal moduli of the composite and the stresses in each material of each element using replacement elements. In this example there are 16 inhomogeneous elements out of a total of 64. Each element is modeled as an 8-node, isoparametric, cubic element. The constituent properties are given in Table 1. The predicted Young's modulus in any of the global coordinate directions of Figure 30 is 0.86×10^6 psi. The corresponding Poisson's ratio is 0.29 and the shear modulus is 0.26×10^6 psi. The peak normal stress concentration in the matrix is 2.5. It occurs at the glass/epoxy interface. The stress concentration at the same point in a continuous fiber reinforced composite with the same ratio of fiber diameter to adjacent fiber spacing is 1.80. The stresses within the constituent materials of the replacement elements appeared to be consistent with the stresses in the neighboring isotropic elements. The distribution of normal stress along two faces of the unit cell is shown in Figure 31.

Sample Problem #7

The last example of the use of the replacement element analysis considers the plain weave unit cell and microgeometry of Figure 32 subjected to uniaxial tension in a reinforcing direction. In this model the resin-impregnated and cured tows are considered to be non-circular tubes of homogeneous orthotropic material that are woven together. These undulating tubes are bonded together at all areas of contact and bonded to the bulk matrix pockets which fill all the interstitial gaps between the tubes. The dimensions of the resin filled tows, the tow spacings and the other geometric details were chosen to best match the microgeometries observed in photomicrographs of woven graphite/epoxy composites (Ref. 1). The analysis was done for the purposes of (a) predicting the extensional stiffness properties of a thick laminate made from symmetrically stacked layers of plain-weave reinforced composite and (b) predicting the detailed stresses and strains within one unit cell of this laminate when it is subject to a simple uniaxial tensile stress in one of the principal tow reinforcing directions.

By the use of structural and load symmetry the essential portion of the unit cell that needs to be analyzed can be reduced in volume and complexity. Figure 32 shows one unit cell of the plain weave microstructure with four planes of symmetry of both load and structure. Only the fraction of unit cell volume between the four planes of symmetry needs to be considered. This enclosed volume is shown in Figure 33 with a set of coordinates that parallel the edges of this regular hexahedron of essential structure. The origin of the coordinates is at the centroid of the hexahedron. These three coordinate axes are also axes of 180° rotational symmetry of both load and structure. Hence, only one quarter of this volume is essential to the analysis. Figure 34 shows this reduced volume which represents only one sixteenth of the original unit cell volume. Further symmetry exists for the structure but not the loading. Figure 34 also shows a simple rectangular finite element mesh superposed on the essential structure. The use of replacement elements permits the application of this grid without much regard for the internal boundaries between the two tow materials and the bulk matrix. The mesh has been graded to give added stress detail near the crossover point of the upper and lower tows (at the origin of Figure 34). The number of finite elements in the smallest essential volume is 64 with 125 node points and 375 degrees of freedom prior to the enforcement of the boundary conditions. Examination of the microstructure within each finite element shows that six, or 9.4% of these elements, contain all three constituent materials. (The two tows are considered to be made from two different materials for bookkeeping convenience.) Fourteen, or 21.9% of the elements, contain only one constituent material. The remaining 44, or 68.7%, contain two constituent materials. This high percentage of replacement elements (78.1%) makes this sample problem different from the previous ones which only required a small number of replacement elements. Another essential difference is the presence of elements containing three constituents. These special elements are treated as follows.

First, note that the two tow materials are in direct contact with each other in each element, rather than being separated by a layer of bulk matrix. Thus, the reinforced portion of each element that contains tow material can be treated as a subelement that contains only two constituent materials. Application of the replacement element logic can then be used to combine these two tow materials into a single anisotropic replacement material. One new factor in this reasoning is that the subelement containing the two constituents is, in general, no longer a right hexahedron. This does not appear to invalidate the replacement process. After both tow materials have been lumped together into a new replacement material then the process can be repeated, combining the new tow replacement material with the bulk matrix material. The only new factor in the latter application of the method is that the combined tow material may be generally anisotropic. This possibility is covered in Appendix C. With these generalizations in place there does not appear to be any reason to prevent the repeated application of the replacement material logic as many times as necessary in

any given element as long as a "tree diagram" of constituent material combinations, as shown in Figure 35A, can be described. Each outer branch ①, ②, ③ of the tree diagram represents a constituent material. Each junction of two materials ④, ⑤ represents an application of the replacement material logic. The lower trunk of the tree diagram ⑥ represents the final replacement material that is used to form the stiffness matrix for the element. The present analysis code (Appendix F) is only general enough to handle the tree diagram of Figure 35A. No more complexity was required for this sample problem.

As examples, consider the microgeometry of a few of the elements from the current sample problem. The element designated [A] in Figure 34 contains only one constituent material, the bulk matrix. The tree diagram for this element is a single trunk of one material with no branches or junctions. No replacement element analysis is required.

The element designated [B] in Figure 34 contains two constituent materials, the bulk matrix and one tow material. Figure 35B isolates this element and shows its tree diagram. The two constituent material branches combine at the single junction to form the trunk material. A single application of the replacement logic suffices for this element. Figure 35C isolates element [C] from Figure 34. This element contains all three constituent materials: the bulk resin and both tow materials. Its tree structure is identical to Figure 35A. The replacement logic is applied to the two tow materials ① and ② at junction ④ initially to form the new material ④. Material ④ and bulk matrix material ③ are then combined at junction ⑤ to form the trunk material ⑥ via the second application of the replacement logic.

Some comments on the complex mixed boundary conditions on the six surfaces of the plain weave structural model are appropriate. Node points on surfaces normal to the z-axis of Figure 36 have the customary symmetry conditions of zero normal displacements* and zero shear forces. The same conditions also apply on the two sides that are at once normal to the x-axis or y-axis but not containing either axis. However, on the two sides containing the coordinate origin the rotational symmetry conditions prevail. Node points along either the x or y-axes cannot displace normal to the axis and must have a zero applied force component along the axis. A node point along either of these two sides (but not on the x or y-axes) must have a corresponding node point that is its mirror image on the opposite side of the coordinate axis that is contained within the side in which the original node point is located (see Figure 36). The tangential displacements at these two image nodes must be the mirror image of each other (across the intervening coordinate axis). The normal displacement must be equal but opposite. The nodal force components normal to the side must be mirror images of each other. The nodal force components parallel to the side must be

* except for rigid body and constant strain displacements

equal but oppositely directed from their mirror image across the coordinate axis. Along the edges of essential structure a combination of the conditions from the intersecting surfaces apply with the displacement conditions prevailing over any contradicting force conditions in any specific coordinate direction. Along the z-axis of Figure 36 the displacements normal to the z-axis vanish, along with the force component parallel to the z-axis. At the coordinate origin all displacements vanish. Corner displacements are determined by the particular strain case being studied, except for displacements conditions at corners A,B,C,D of Figure 36. There the aforementioned mixed rotational symmetry conditions apply to forces and displacements normal to the faces containing the x or y-axis.

Table 4 contains all the geometric information required for each element. These values were all obtained by viewing composite photomicrographs and making many sketches of planar cuts through the essential structure. It is a chore that would lend itself well to preprocessing. However, it is a matter of only a few days work as opposed to the weeks of work associated with setting up and checking out a finite element mesh based upon homogeneous elements.

The tow composite properties used are typical of unidirectional, intermediate modulus, graphite/epoxy prepreg. Most prepregs cure out to about 65% fiber volume fraction. The fiber volume fraction within a tow of a fabric reinforced composite is generally in the 70% to 75% range. This could justify using higher tow composite moduli in the analysis. However, the loss in properties due to the weaving process have never been established. The use of the lower properties (associated with 65% fiber volume fraction) is an attempt to compensate for fiber breakage, misalignment, and other weaving and processing damage. The overall fiber volume fraction for the analysis model was 64% with 15% interstitial bulk matrix volume fraction and 85% tow volume fraction. The constituent material properties correspond to the epoxy properties of Table 1 and the graphite/epoxy A properties of Table 2. The predicted extensional elastic constants are, with reference to the coordinates of Figure 33,

$$\begin{aligned} E_x &= E_y = 7.88 \times 10^6 \text{ psi} \\ E_z &= 1.69 \times 10^6 \text{ psi} \\ \nu_{xz} &= \nu_{yz} = 0.321 \\ \nu_{xy} &= \nu_{yx} = 0.048 \end{aligned}$$

As a reference point, the moduli from test data reported in Ref. 1 are

$$\begin{aligned} E_x &= 9.13 \times 10^6 \text{ psi (warp)} \\ E_y &= 8.83 \times 10^6 \text{ psi (fill)} \\ \nu_{xy} &= 0.11 \end{aligned}$$

With conventional laminate theory, for a cross-ply laminate with a 15% thick layer of bulk resin, the result would be

$$E_x = E_y = 9.68 \times 10^6 \text{ psi}$$
$$\nu_{xy} = 0.050$$

The stress results are more interesting than the moduli predictions. For a unit remotely applied stress in the warp direction, with all other average stresses held at zero, the peak warp tow stress has a value of about 4, giving a stress concentration factor of the same amount. This stress occurs inside rather than on the surface of the unit cell and away from the cross over point of the adjacent tows. It occurs as a result of high bending plus axial strain in the tow that roughly parallels the load direction. Figure 37 contains contour plots of the stress in the fiber direction on the primary load carrying tow surface. The axial stress in the fill tows are insignificant. Figure 37 also contains a plot of the axial fiber strain concentration factors based on the ratio of fiber longitudinal strain divided by average composite strain in the load direction. These values differ significantly from results reported in Ref. 9. The peak fiber strain concentration from the current analysis is about 1.5 compared to 2.6 reported in Ref. 9. Also, the location of the peak strains do not coincide. The peak strain occurs on the curved portion of the tow surface away from the edges of the tow and away from the inflection point of the principal axis of the tow. In Ref. 9 it occurs at the edges of the tow at the adjacent tow cross-over point. Plots of the other stress and strain components also differ significantly. The two sets of analyses should not be duplicates of each other because there were various differences in the models, the constituent properties, the degree of mesh refinement, the order of the elements, etc. However, the differences in the results seem larger than expected. Differences in tow cross-sectional variation along the tow axis may account for much of the discrepancy. In the current analysis very little tow thickness variation was permitted because very little was seen in composite photomicrographs. However, in Ref. 9 significant necking of the tow thickness (at the sides of the tow) was built into the analysis model near the tow crossover point. Some of the strain concentrations could have been the result of these differences in cross-sectional modeling.

In summary, the stress predictions for the sample problem appear to adequately reflect all the major combined bending, stretching and shearing effects that were anticipated in the plane weave tension analysis. The causes for some of the local strain differences between this analysis and that of Ref. 9 remain to be resolved.

The rotational symmetry boundary conditions that were used with this sample problem are not used frequently and were not included in the computer program listed in Appendix F. They were used in this problem simply to avoid the necessity of inverting stiffness matrices larger than 300 square. The program in Appendix F has the more common conditions of geometric unit cell

surface symmetry plus load symmetry (and asymmetry) built into it. The same results could have been obtained using the program in Appendix F with some of the larger array dimensions increased four fold, and one quarter of the unit cell volume analyzed rather than one sixteenth of the volume.

V. Conclusions

The three-dimensional elastic analysis of complex composite microstructures is made difficult by the constraint imposed by conventional finite element analysis on the correspondence of internal material interfaces and element boundaries. The concept of a replacement element is introduced for the purpose of relaxing this constraint. The replacement element combines the constituent materials within an inhomogeneous element into a single anisotropic material to which the established finite element procedures may be applied. This constituent material combination depends on simple composite mechanics models for parallel bonded plates. This procedure involves a physical rearrangement of the materials within the element and therefore represents an idealization or approximation of the true material interactions. It has been shown that the use of these replacement elements can incur errors on the order of 20% in the predicted stresses within the constituents. However, in the more complex problems in which the replacement elements occur less frequently the errors in stiffness and internal stress predictions appear to be within a range that is acceptable for some engineering applications; namely, trade-off studies that lead to the ranking or selection of specific reinforcement microgeometries to meet specific structural requirements.

Through the use of several example problems of increasing complexity both the application and results of the replacement element method are observed. The application is simpler and easier than the conventional finite element method in complicated 3-D problems such as those posed by many fabric reinforced composite microgeometries. The results are less accurate and less reliable, but still acceptable, in view of the statistical variation in unit cell microgeometries and their boundary conditions. A large number of finite elements are still required to model a complex microstructure but beyond that point the mechanical analysis is much easier to automate and eventually merge with computerized unit cell microgeometry generators, preprocessors and postprocessors. The use of replacement elements still requires some skill in the selection of rectangular grids which minimize both the number and complexity of the replacement elements.

It remains to establish guidelines for the use of replacement elements so as to minimize the approximation errors, and also to improve upon the process itself to make it more sensitive to the details of the constituent material distribution within an element. The latter tasks could not be undertaken within the seven man-month scope of this effort.

References

1. Foye, R.L., "Finite Element Analysis of the Stiffness of Fabric Reinforced Composites", NASA Contractor Report No. CR-189597 (January 1982).
2. Foye, R.L., "Thermal Expansion of Fabric Reinforced Composites", Fiber-Tex 90, NASA Conference Publication 3128 (August 1990).
3. Bathe, Klaus-Jurgen, "Finite Element Procedures in Engineering Analysis". Prentice-Hall Inc., Englewood Cliffs, NJ (1982)
4. Jones, Robert M., "Mechanics of Composite Materials", McGraw-Hill Book Co., Washington, D.C. (1975).
5. Foye, R.L. and Baker, D.J., "Design/Analysis Methods for Advanced Composite Structures", Air Force Materials Lab. report No. AFML-TR-70-299 (February 1971).
6. Harris, Charles E. and Lee, Jong-Won, "A Micromechanics Model of the Stiffness and Strength of Laminates with Fiber Waviness", NASA Contractor Report 181670 (July 1988).
7. "MSC/NASTRAN - Handbook for Linear Static Analysis", MSC NASTRAN Version 61, The MacNeal-Schwendler Corporation, Los Angeles, CA (December 1981).
8. Jortner, Jules, "Effect of Weave Geometry and Yarn Waviness on Thermomechanical Properties of Multi-Directional Composites", 3-D Composite Materials Conference, NASA Conference Publication 2420 (November 1985).
9. Whitcomb, John D., "Three Dimensional Stress Analysis of Plain Weave Composites", Composite Materials: Fatigue and Fracture (3rd Vol.), ASTM STP 1110, T.K. O'Brien, Ed., American Society for Testing and Materials, Philadelphia, PA (1991), pp 417-438.

Table 1: Isotropic Constituents

	$E \times 10^6 \text{psi}$	ν	$G \times 10^6 \text{psi}$
Aluminum	10.0	0.30	4.0
E Glass	10.0	0.25	4.0
Epoxy	0.5	0.35	0.18

Table 2: Orthotropic Constituents

	$E_1 \times 10^6 \text{psi}$	$E_2, E_3 \times 10^6 \text{psi}$	ν_{12}, ν_{13}	ν_{23}
GR/EP A	18.0	1.5	0.23	0.35
GR/EP B	21.0	1.7	0.23	0.30

	$G_{12}, G_{13}, \times 10^6 \text{psi}$	$G_{23} \times 10^6 \text{psi}$
GR/EP A	0.7	0.7
GR/EP B	0.7	0.7

Table 3: Microgeometry Data for Sample Problem #6

Element No. (I,J,K)	Interfacial Normal Δ		Volume Fractions	
	Ψ_1	Ψ_2	V_{GL}	V_{EP}
1, 1, 1	-	-	1.0	0.0
1, 1, 2	-	-	1.0	0.0
1, 1, 3	45°	78°	0.57	0.43
1, 1, 4	-	-	0.0	1.0
1, 2, 1	-	-	1.0	0.0
1, 2, 2	70°	45°	0.90	0.10
1, 2, 3	70°	57°	0.15	0.85
1, 2, 4	-	-	0.0	1.0
1, 3, 1	80°	10°	0.57	0.43
1, 3, 2	80°	30°	0.15	0.85
1, 3, 3	-	-	0.0	1.0
1, 3, 4	-	-	0.0	1.0
1, 4, 1	-	-	0.0	1.0
1, 4, 2	-	-	0.0	1.0
1, 4, 3	-	-	0.0	1.0
1, 4, 4	-	-	0.0	1.0
2, 1, 1	-	-	1.0	0.0
2, 1, 2	20°	45°	0.90	0.10
2, 1, 3	20°	57°	0.15	0.85
2, 1, 4	-	-	0.0	1.0
2, 2, 1	45°	14°	0.90	0.10
2, 2, 2	45°	45°	0.55	0.45
2, 2, 3	-	-	0.0	1.0
2, 2, 4	-	-	0.0	1.0
2, 3, 1	57°	8°	0.15	0.85
2, 3, 2	-	-	0.0	1.0
2, 3, 3	-	-	0.0	1.0
2, 3, 4	-	-	0.0	1.0
2, 4, 1	-	-	0.0	1.0
2, 4, 2	-	-	0.0	1.0
2, 4, 3	-	-	0.0	1.0
2, 4, 4	-	-	0.0	1.0

Table 3: Microgeometry Data for Sample Problem #6
(continued)

Element No. (I,J,K)	Interfacial Normal \angle		Volume Fractions	
	Ψ_1	Ψ_2	V_{GL}	V_{EP}
3, 1, 1	10°	10°	0.57	0.43
3, 1, 2	10°	30°	0.15	0.85
3, 1, 3	-	-	0.0	1.0
3, 1, 4	-	-	0.0	1.0
3, 2, 1	33°	8°	0.15	0.85
3, 2, 2	-	-	0.0	1.0
3, 2, 3	-	-	0.0	1.0
3, 2, 4	-	-	0.0	1.0
3, 3, 1	-	-	0.0	1.0
3, 3, 2	-	-	0.0	1.0
3, 3, 3	-	-	0.0	1.0
3, 3, 4	-	-	0.0	1.0
3, 4, 1	-	-	0.0	1.0
3, 4, 2	-	-	0.0	1.0
3, 4, 3	-	-	0.0	1.0
3, 4, 4	-	-	0.0	1.0
4, 1, 1	-	-	0.0	1.0
4, 1, 2	-	-	0.0	1.0
4, 1, 3	-	-	0.0	1.0
4, 1, 4	-	-	0.0	1.0
4, 2, 1	-	-	0.0	1.0
4, 2, 2	-	-	0.0	1.0
4, 2, 3	-	-	0.0	1.0
4, 2, 4	-	-	0.0	1.0
4, 3, 1	-	-	0.0	1.0
4, 3, 2	-	-	0.0	1.0
4, 3, 3	-	-	0.0	1.0
4, 3, 4	-	-	0.0	1.0
4, 4, 1	-	-	0.0	1.0
4, 4, 2	-	-	0.0	1.0
4, 4, 3	-	-	0.0	1.0
4, 4, 4,	-	-	0.0	1.0

Table 4: Microgeometry Data for Sample Problem #7

ELEM. NO.	TOW MATERIAL①				TOW MATERIAL②				MATRIX MATERIAL③			TOW/TOW INTERFACE NORMAL \angle		TOW/MATRIX INTERFACE NORMAL \angle	
	ν_f	ϕ_1	ϕ_2		ν_f	ϕ_1	ϕ_2		ν_f	ϕ_1	ϕ_2	ψ_1	ψ_2	ψ_1	ψ_2
1, 1, 1	0	0	5.0		0	90.0	0		1.00	0.0	0.0	-	-	-	-
1, 1, 2	.10	↑	↑		.45	↑	-5.0		.45	↑	↑	-45.0	97.0	45.0	83.0
1, 1, 3	.45				.10		-5.0		.45			-45.0	97.0	45.0	97.0
1, 1, 4	0				0		0		1.00			-	-	-	-
1, 2, 1	0				.05		-3.0		.95			-	-	31.0	84.2
1, 2, 2	.35				.50		-3.0		.15			-31.0	95.8	31.0	84.2
1, 2, 3	.85				0		-3.0		.15			-	-	31.0	95.8
1, 2, 4	.05				0		-3.0		.95			-	-	31.0	95.8
1, 3, 1	0				.25		-1.0		.75			-	-	12.0	84.8
1, 3, 2	.63				.35		-1.0		.02			12.0	84.8	-12.0	95.2
1, 3, 3	.98				0		-1.0		.02			-	-	12.0	95.2
1, 3, 4	.25				0		-1.0		.75			-	-	12.0	95.2
1, 4, 1	0				.30		0		.70			-	-	0	85.0
1, 4, 2	.70	↓	↓		.30	↓	0		0	↑	↓	0	95.0	-	0
1, 4, 3	1.00				0		0		0			-	-	-	-
1, 4, 4	.30	0	5.0		0	90.0	0		.70	0.0	0.0	-	-	0	95.0

Table 4: Microgeometry Data for Sample Problem #7 (continued)

ELEM. NO.	TOW MATERIAL①			TOW MATERIAL②			MATRIX MATERIAL③			TOW/TOW INTERFACE NORMAL \angle		TOW/MATRIX INTERFACE NORMAL \angle	
	v_f	ϕ_1	ϕ_2	v_f	ϕ_1	ϕ_2	v_f	ϕ_1	ϕ_2	ψ_1	ψ_2	ψ_1	ψ_2
I, J, K													
2, 1, 1	0	0	3.0	.05	90.0	-5.0	.95	0	0	-	-	59.0	84.2
2, 1, 2	0	↑	↑	.85	↑	-5.0	.15	↑	↑	-	-	59.0	84.2
2, 1, 3	.50			.35		-5.0	.15			-59.0	95.8	59.0	95.8
2, 1, 4	.05			0		-5.0	.95			-	-	59.0	95.8
2, 2, 1	0			.25		-3.0	.75			-	-	45.0	85.8
2, 2, 2	.08			.92		-3.0	0			-45.0	94.2	-	-
2, 2, 3	.92			.08		-3.0	0			-45.0	94.2	-	-
2, 2, 4	.25			0		-3.0	.75			-	-	45.0	94.2
2, 3, 1	0			.50		-1.0	.50			-	-	18.5	86.8
2, 3, 2	.25			.75		-1.0	0			-18.5	93.2	-	-
2, 3, 3	1.00			0		-1.0	0			-	-	-	-
2, 3, 4	.50			0		-1.0	.50			-	-	-18.5	93.2
2, 4, 1	0	↓	↓	.70	↓	0	.30	↓	↓	-	-	0	87.0
2, 4, 2	.30			.70		0	0			0	93.0	-	0
2, 4, 3	1.00			0		0	0			-	-	-	-
2, 4, 4	.70	0	3.0	0	90.0	0	.30	0	0	-	-	0	93.0

Table 4: Microgeometry Data for Sample Problem #7 (continued)

ELEM. NO.	TOW MATERIAL①				TOW MATERIAL②				MATRIX MATERIAL③			TOW/TOW INTERFACE NORMAL Δ		TOW/MATRIX INTERFACE NORMAL Δ	
	v_f	ϕ_1	ϕ_2	v_f	ϕ_1	ϕ_2	v_f	ϕ_1	ϕ_2	v_f	ϕ_1	ψ_1	ψ_2	ψ_1	ψ_2
I, J, K															
3, 1, 1	0	0	1.0	.25	90.0	-5.0	.75	0	0			-	-	78.0	84.8
3, 1, 2	0	↑	↑	.98	↑	-5.0	.02	↑	↑			-	-	78.0	84.8
3, 1, 3	.35			.63		-5.0	.02					-78.0	95.2	78.0	95.2
3, 1, 4	.25			0		-5.0	.75					-	-	78.0	95.2
3, 2, 1	0			.50		-3.0	.50					-	-	71.5	86.8
3, 2, 2	0			1.00		-3.0	0					-	-	-	-
3, 2, 3	.75			.25		-3.0	0					-71.5	93.2	-	-
3, 2, 4	.50			0		-3.0	.50					-	-	71.5	93.2
3, 3, 1	0			.80		-1.0	.20					-	-	45.0	88.5
3, 3, 2	.05			.95		-1.0	0					-45.0	91.5	-	-
3, 3, 3	.95			.05		-1.0	0					-45.0	91.5	-	-
3, 3, 4	.80			0		-1.0	.20					-	-	0	89.0
3, 4, 1	0	↓	↓	.90	↓	0	.10	↓	↓			-	-	0	89.0
3, 4, 2	.10			.90		0	0					0	91.0	-	-
3, 4, 3	1.00			0		0	0					-	-	-	-
3, 4, 4	.90	0	1.0	0	90.0	0	.10	0	0			-	-	0	91.0

Table 4: Microgeometry Data for Sample Problem #7 (continued)

ELEM. NO.	TOW MATERIAL①			TOW MATERIAL②			MATRIX MATERIAL③			TOW/TOW INTERFACE NORMAL Δ		TOW/MATRIX INTERFACE NORMAL Δ	
	v_f	ϕ_1	ϕ_2	v_f	ϕ_1	ϕ_2	v_f	ϕ_1	ϕ_2	ψ_1	ψ_2	ψ_1	ψ_2
I, J, K													
4, 1, 1	0	0	0	.30	90.0	-5.0	.70	0	0	-	-	90.0	85.0
4, 1, 2	0	↑	↑	1.00	↑	-5.0	0	↑	↑	-	-	-	-
4, 1, 3	.30			.70		-5.0	0			90.0	85.0	-	-
4, 1, 4	.30			0		-5.0	.70			-	-	-90.0	85.0
4, 2, 1	0			.70		-3.0	.30			-	-	90.0	87.0
4, 2, 2	0			1.00		-3.0	0			-	-	-	-
4, 2, 3	.70			.30		-3.0	0			90.0	87.0	-	-
4, 2, 4	.70			0		-3.0	.30			-	-	-90.0	87.0
4, 3, 1	0			.90		-1.0	.10			-	-	90.0	89.0
4, 3, 2	0			1.00		-1.0	0			-	-	-	-
4, 3, 3	.90			.10		-1.0	0			90.0	89.0	-	-
4, 3, 4	.90			0		-1.0	.10			-	-	-90.0	89.0
4, 4, 1	0	↑	↑	1.00	↑	0	0	↑	↑	-	-	-	-
4, 4, 2	0			1.00		0	0			-	-	-	-
4, 4, 3	1.00			0		0	0			-	-	-	-
4, 4, 4	1.00	0	0	0	90.0	0	0	0	0	-	-	-	-

Table 5: Resident Constituent Material Properties

MATERIAL	E_1 ($\times 10^6$ psi)	E_2, E_3 ($\times 10^6$ psi)	ν_{12}, ν_{13}	ν_{23}	G_{12}, G_{13} ($\times 10^6$ psi)	G_{23} ($\times 10^6$ psi)	ϵ_1 (ten.)	ϵ_1 (comp.)	ϵ_2, ϵ_3 (ten.)	ϵ_2, ϵ_3 (comp.)
1 GR/EP Low mod.	18.0	1.5	0.23	0.30	0.7	0.7	0.01	0.01	0.01	0.01
2 GR/EP Med. mod.	21.0	1.7	0.23	0.30	0.7	0.7	0.01	0.01	0.01	0.01
3 GR/EP Hi. Mod.	30.0	1.7	0.23	0.30	0.7	0.07	0.01	0.01	0.01	0.01
4 GL/EP Low Mod.	10.0	1.5	0.25	0.35	0.7	0.7	0.01	0.01	0.01	0.01
5 GL/EP Hi. Mod.	12.0	1.5	0.25	0.35	0.7	0.7	0.01	0.01	0.01	0.01
6 EPOXY MATRIX	0.5	0.5	0.35	0.35	0.18	0.18	0.01	0.01	0.01	0.01
7 ALUMINUM	10.0	10.0	0.30	0.30	4.0	4.0	0.01	0.01	0.01	0.01
8 GLASS FILLER	10.0	10.0	0.25	0.25	4.0	4.0	0.01	0.01	0.01	0.01

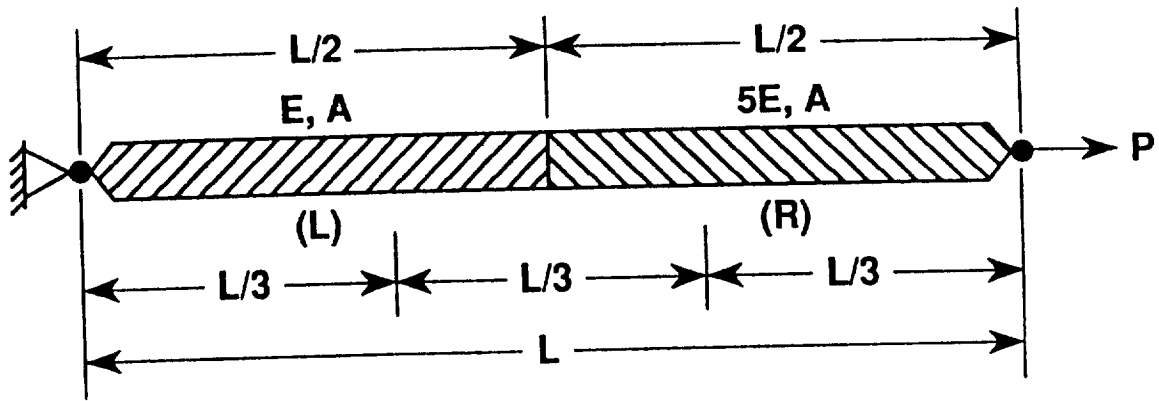


Figure 1. Tension bar sample problem #1.

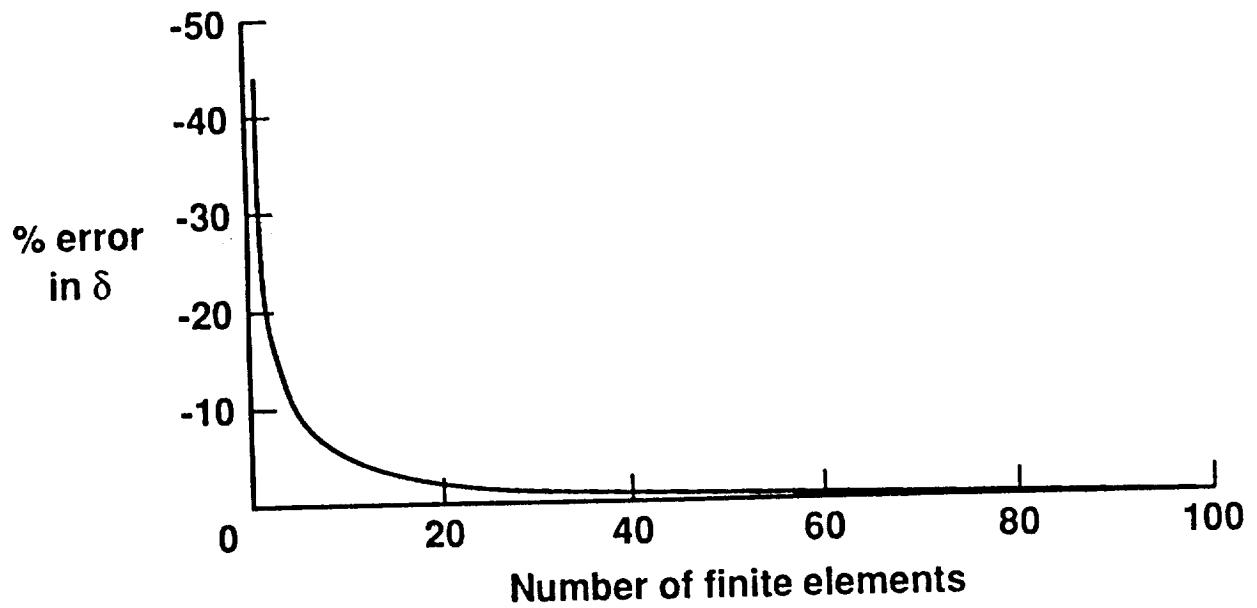


Figure 2. Inhomogeneous element error in tension bar elongation.

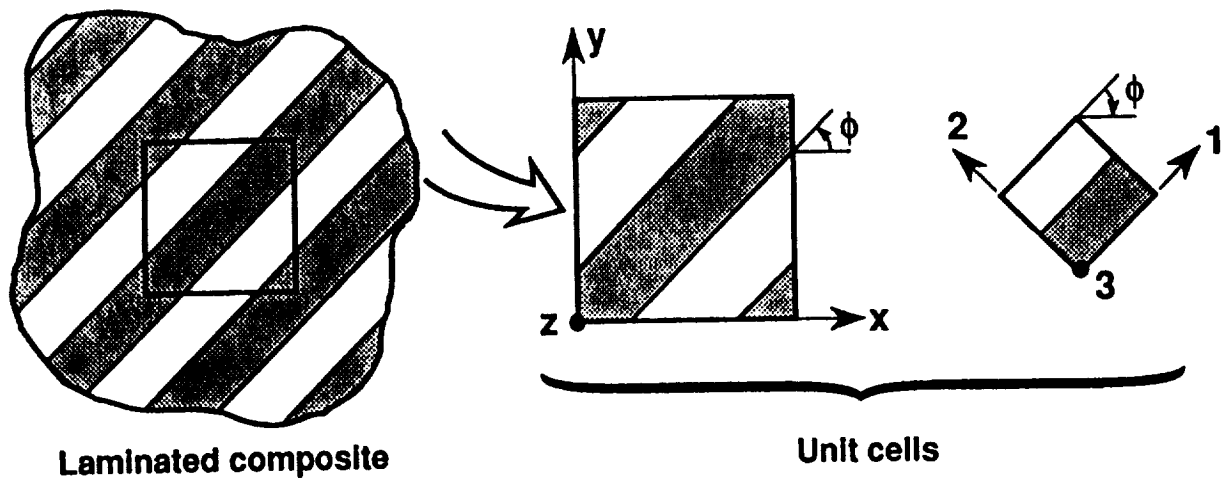


Figure 3. Sample problem #2 microgeometry.

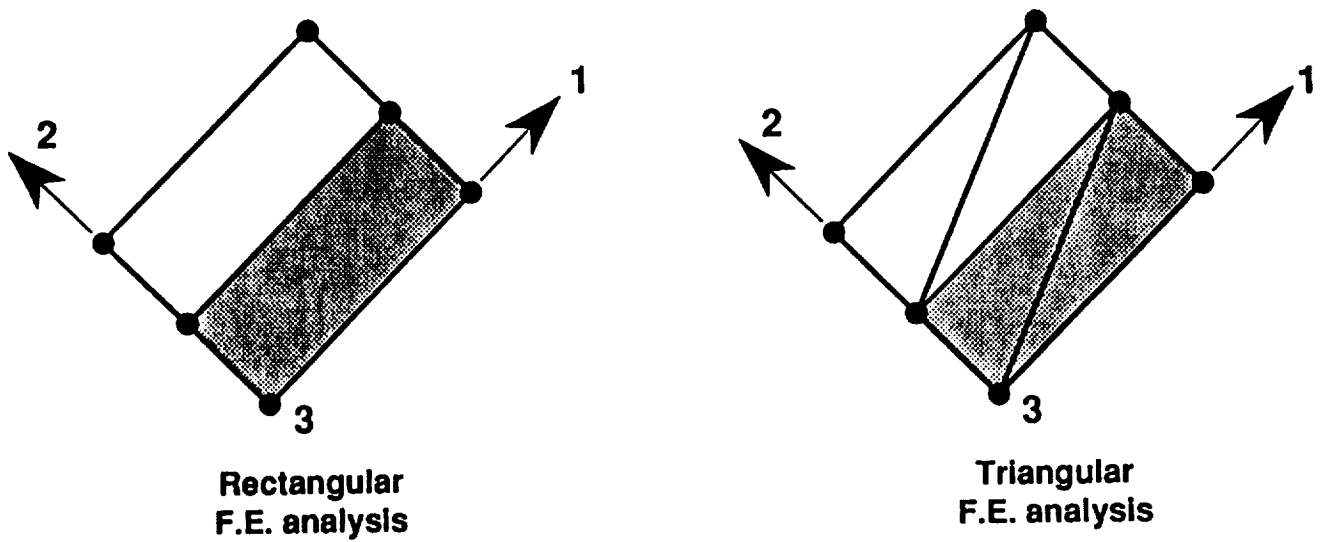


Figure 4. F.E. grids for sample problem #2 in principal coordinates.

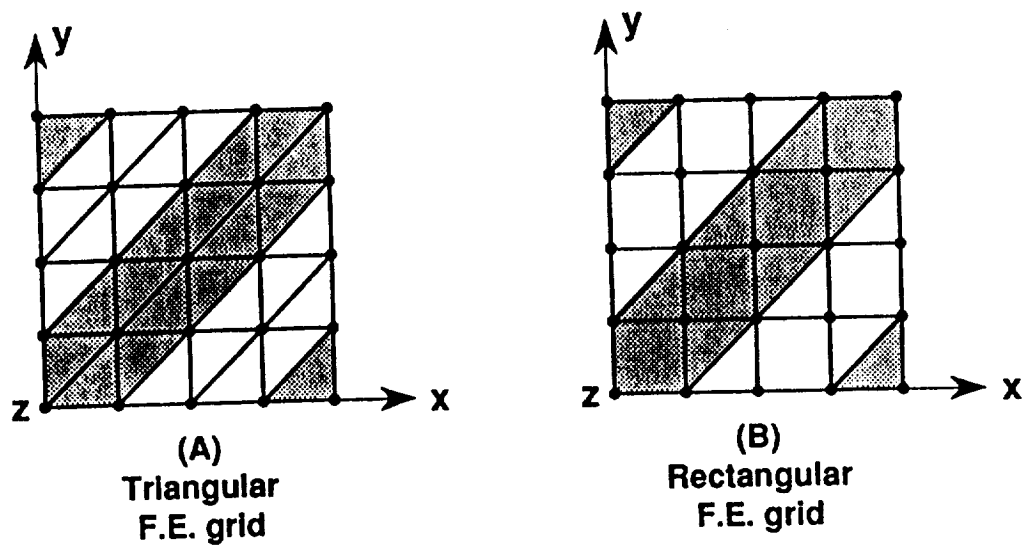


Figure 5. F.E. grids for sample problem #2 in global coordinates.

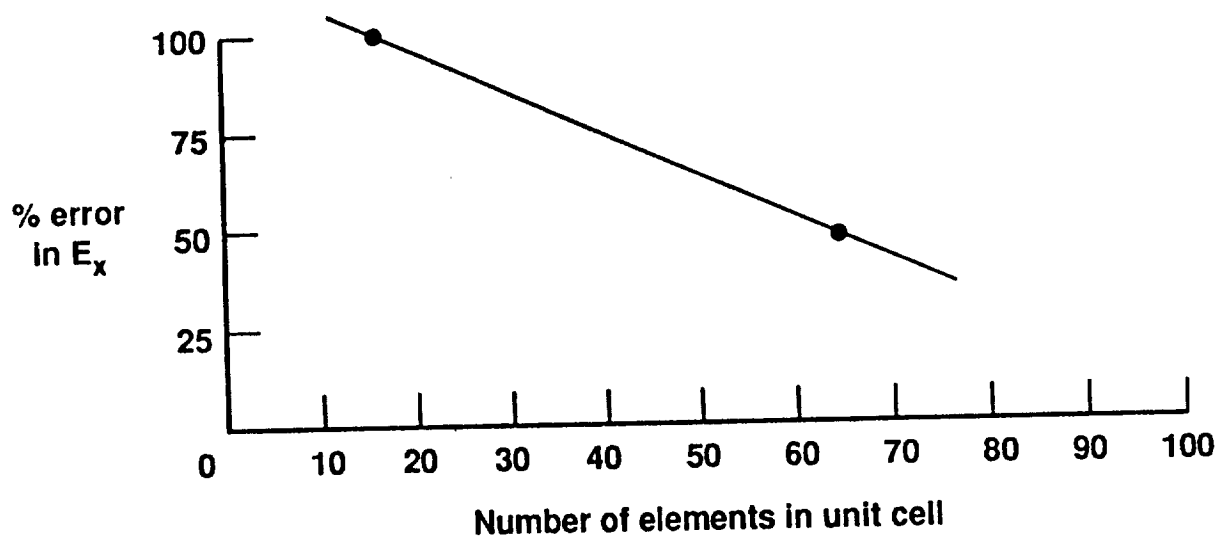


Figure 6. Convergence of rectangular inhomogeneous F.E. solution.

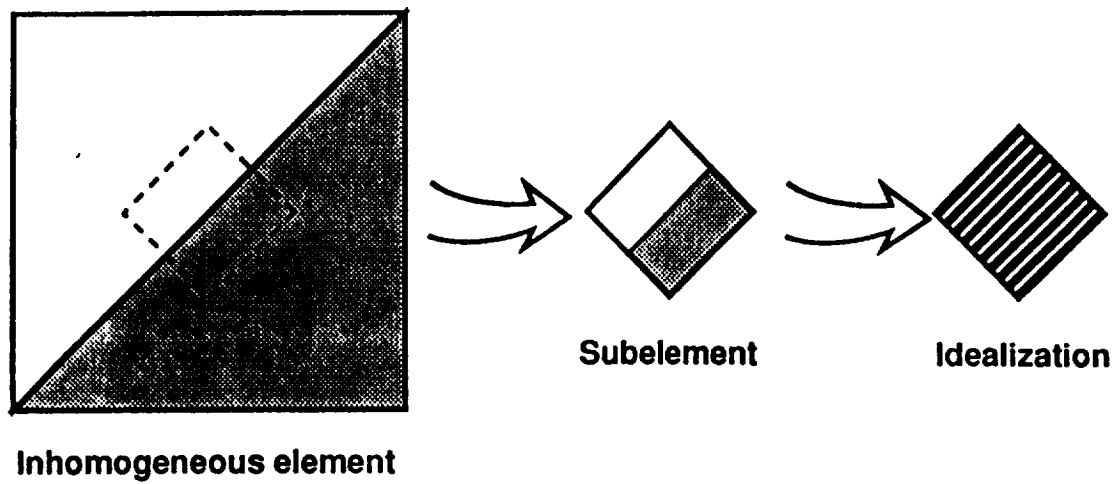


Figure 7. 2-D Idealization of Inhomogeneous element.

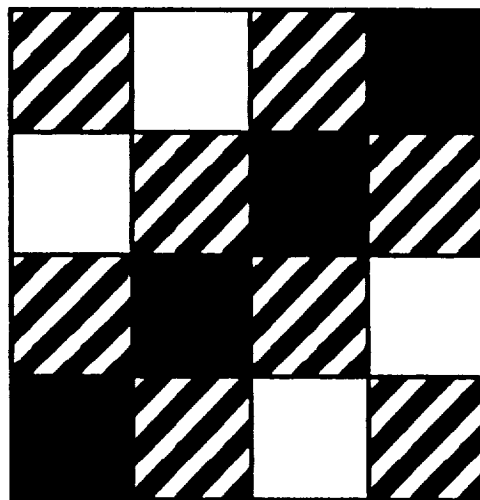


Figure 8. Unit cell Idealization for sample problem #2.

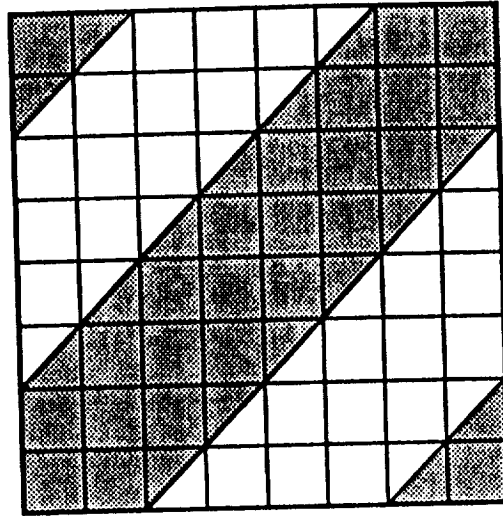
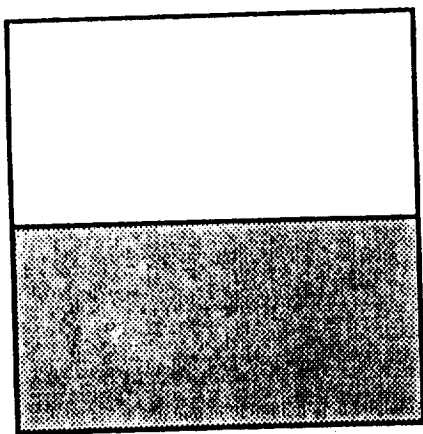
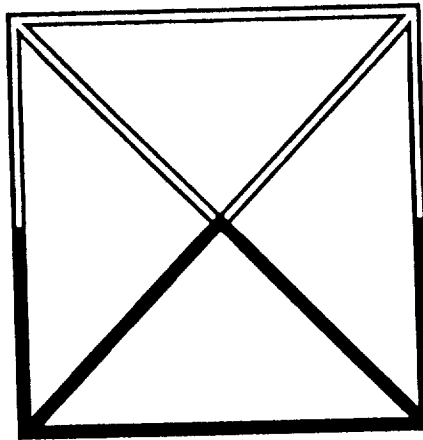


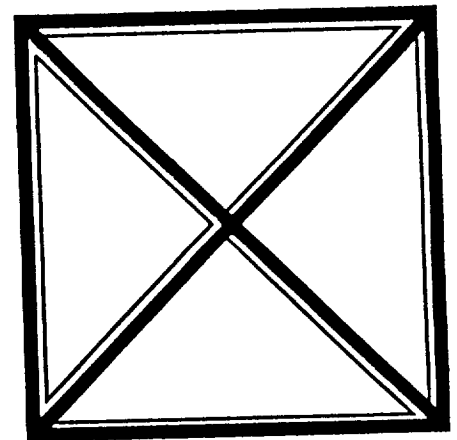
Figure 9. Finer F.E. grid for unit cell analysis.



(A)
Unit cell



(B)
Preferred lattice model



(C)
Energy formula model

Figure 10. Lattice model of unit cell.

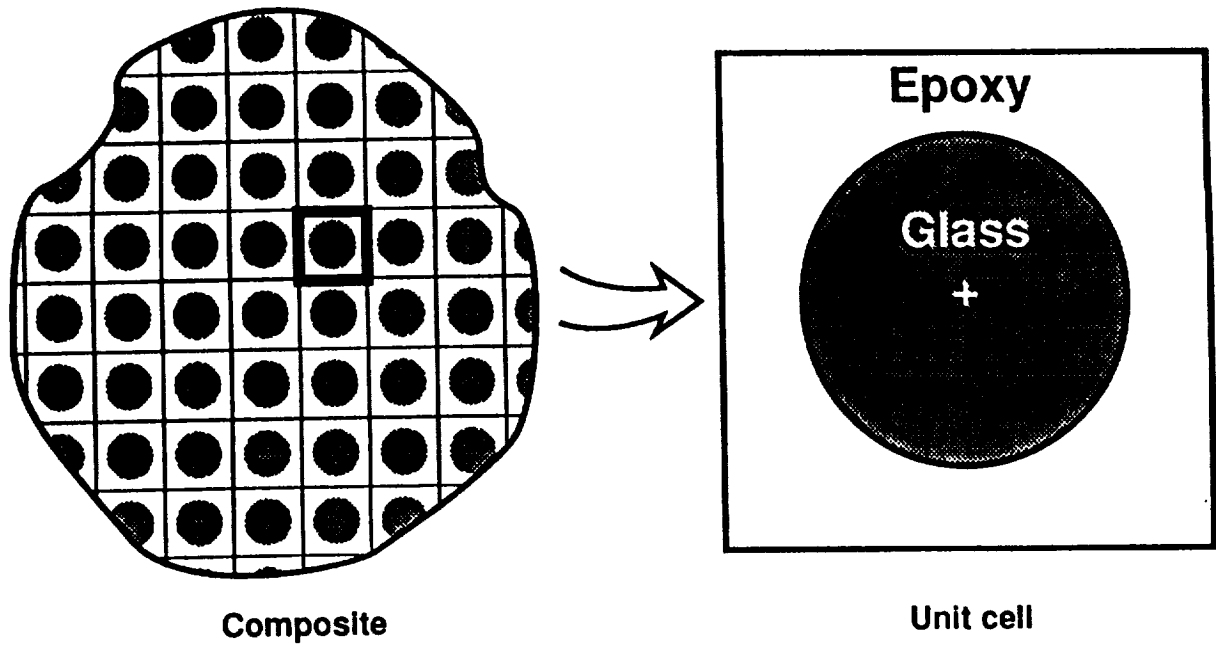


Figure 11. Unidirectional glass/epoxy composite.

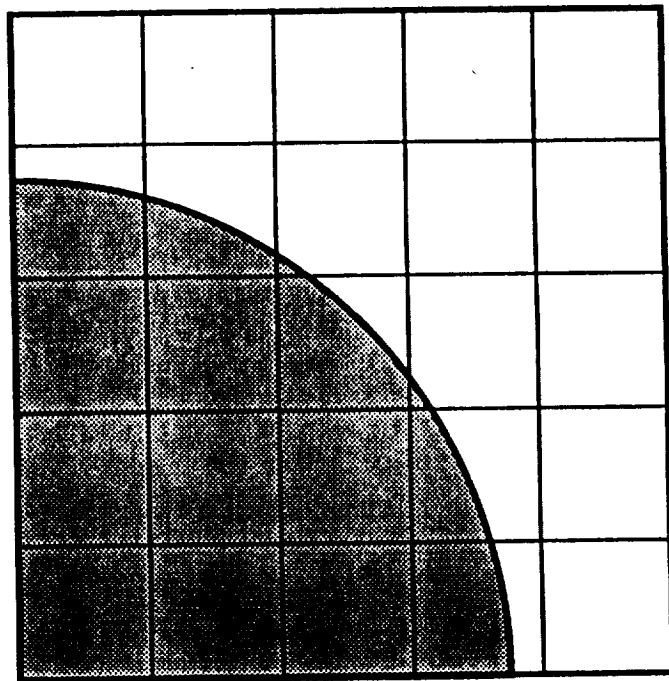


Figure 12. F.E. grid for unidirectional composite.

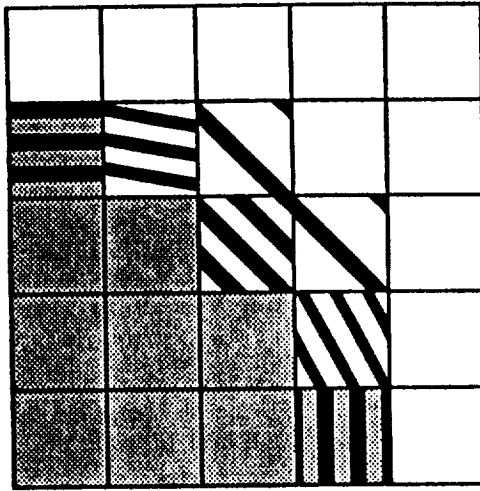


Figure 13. Unit cell idealization for sample problem #3.

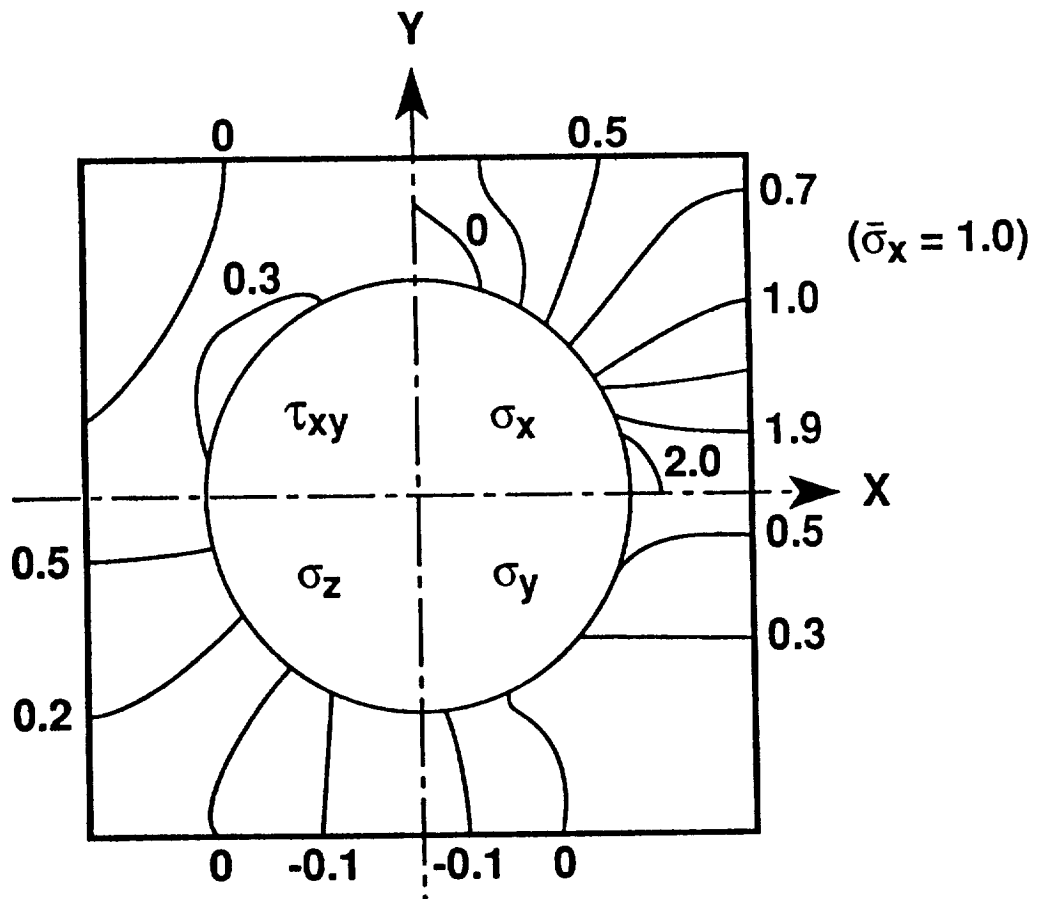


Figure 14. Contour plots of matrix stresses due to unit average stress in the x-direction.

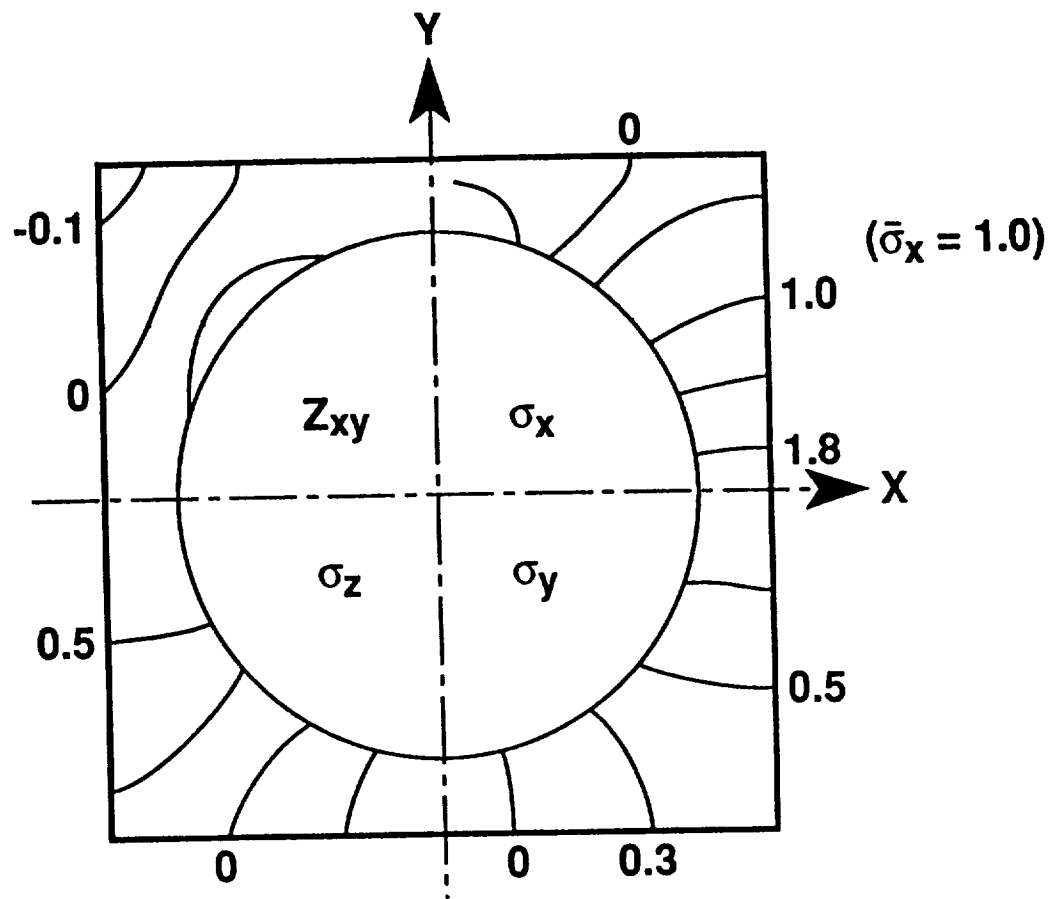


Figure 15. Matrix stress contours from conventional F.E. analysis.

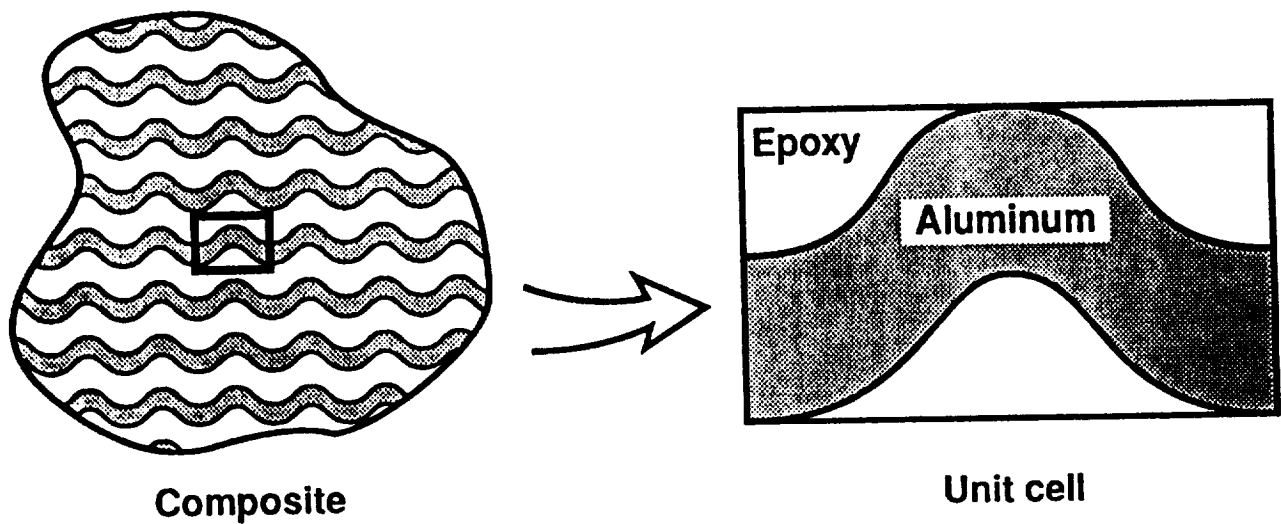


Figure 16. Microgeometry for sample problem #4.

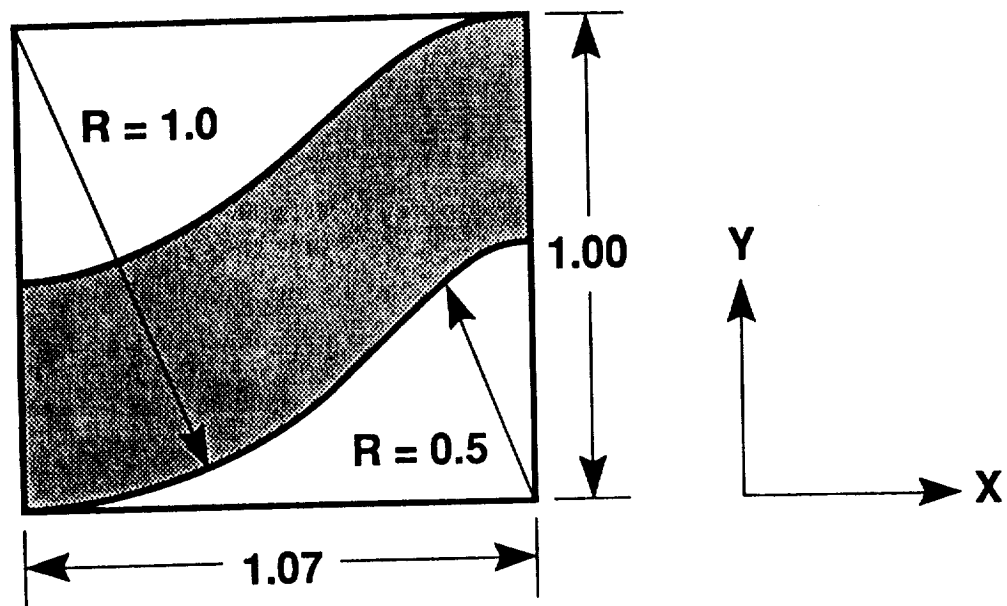


Figure 17. Microstructural dimensions for sample problem #4.

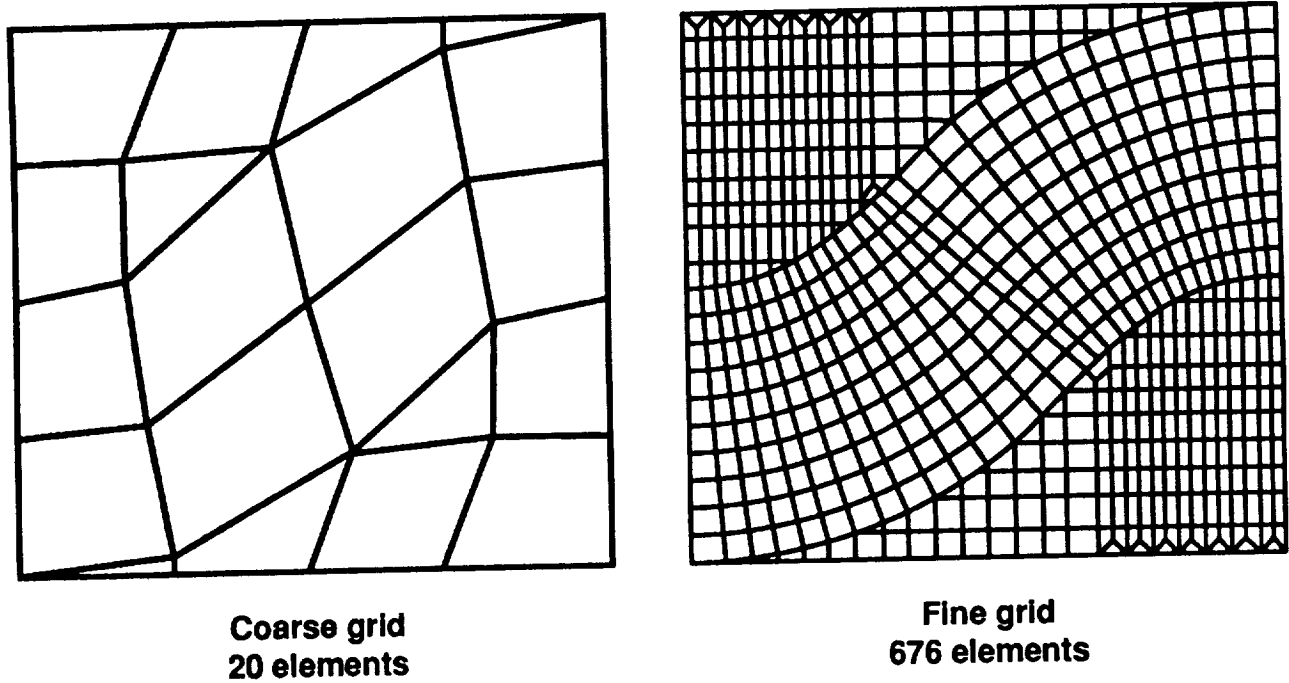


Figure 18. F.E. grids for NASTRAN analysis of sample problem #4.

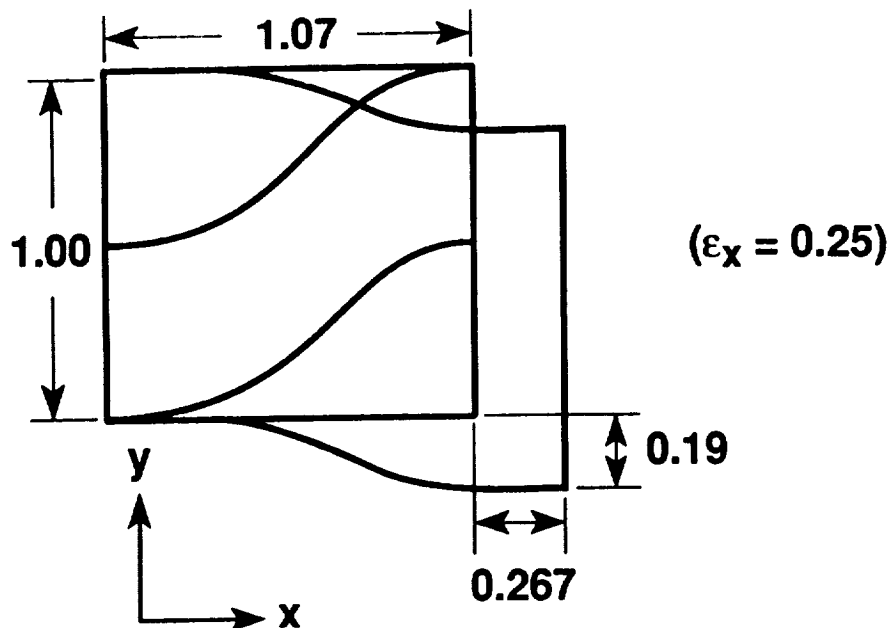


Figure 19. Surface deflections of unit cell from NASTRAN.

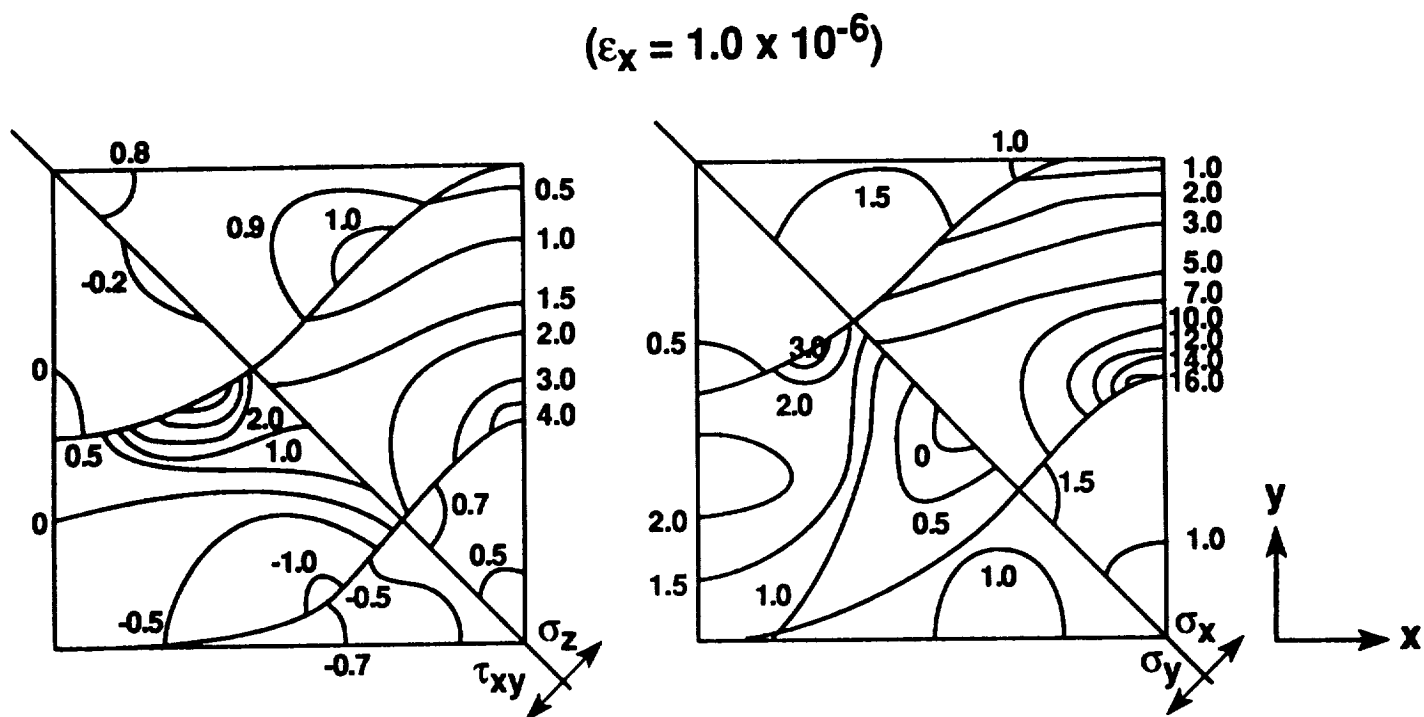


Figure 20. Baseline internal stress contours from NASTRAN.

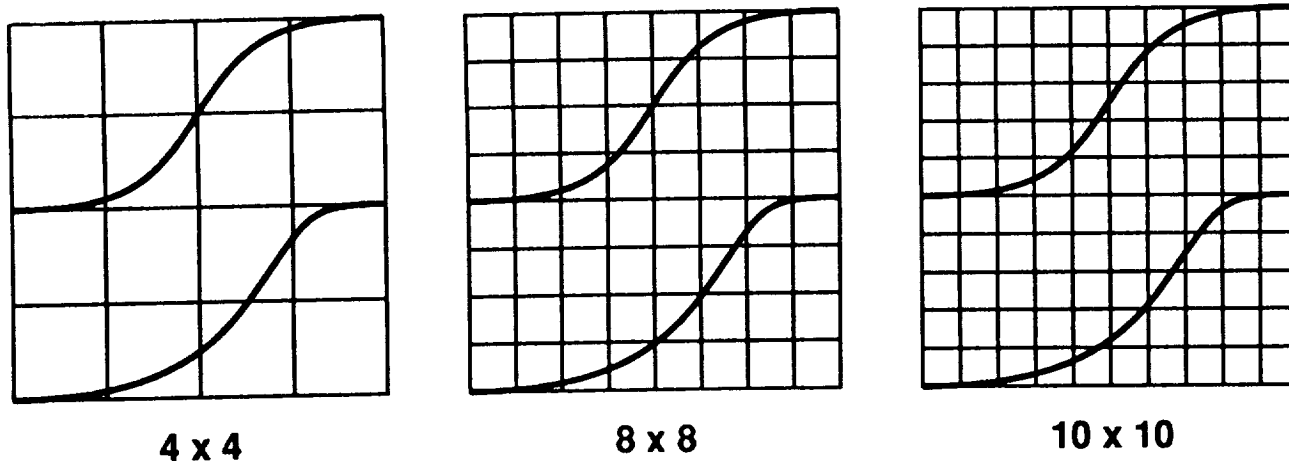


Figure 21. F.E. grids applied to sample problem #4.

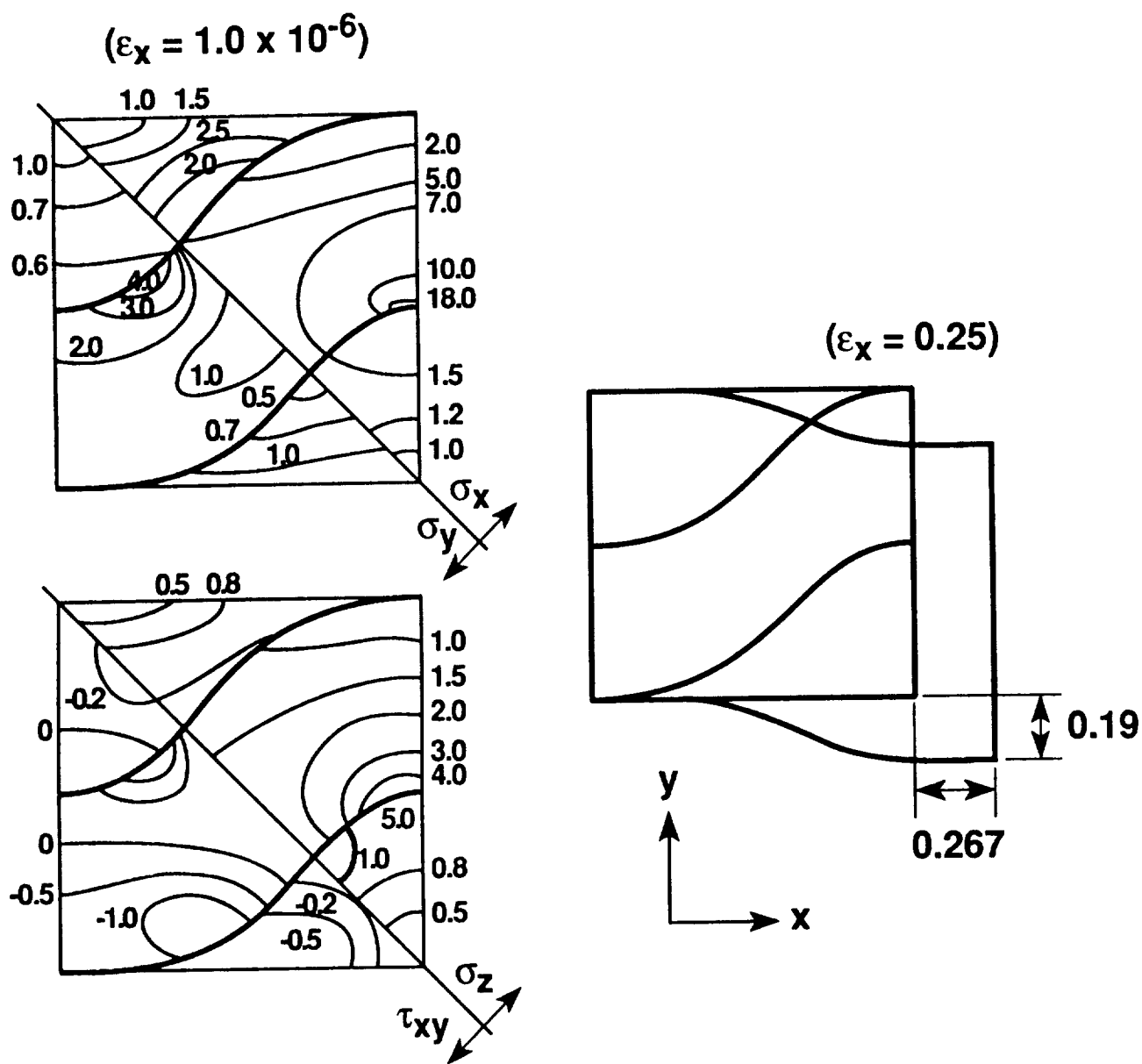


Figure 22. Stress/deflection results from replacement elements.

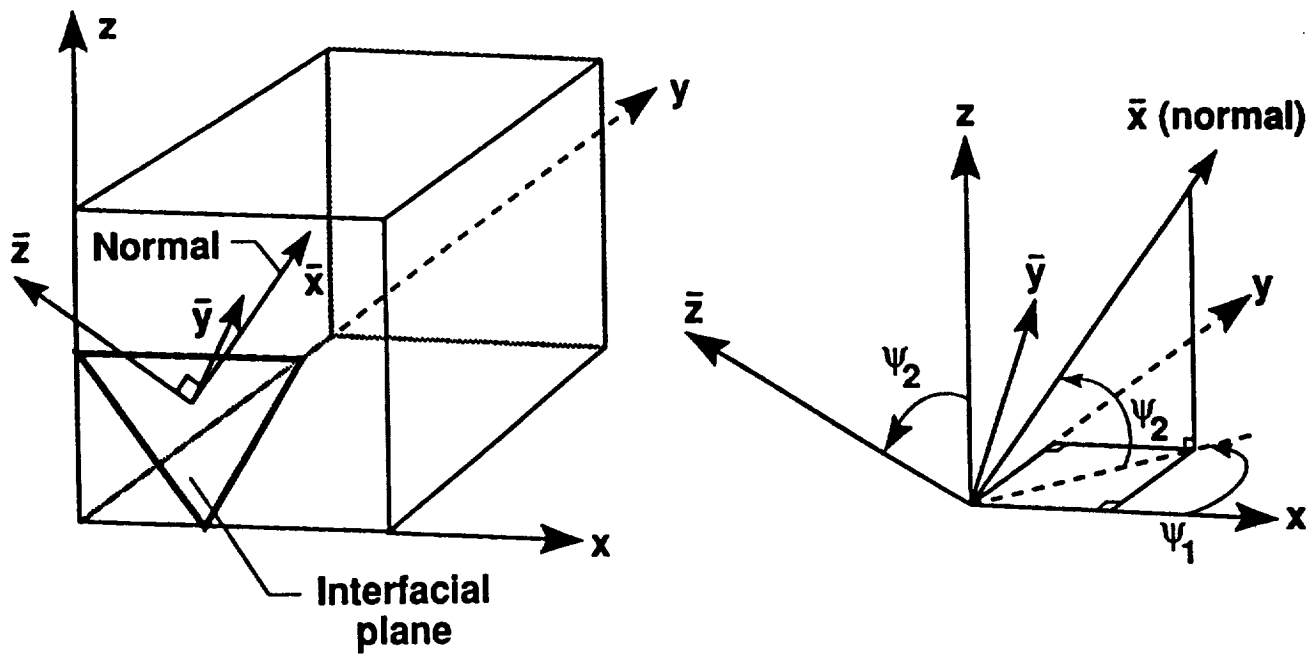


Figure 23. Global and interfacial coordinate systems.

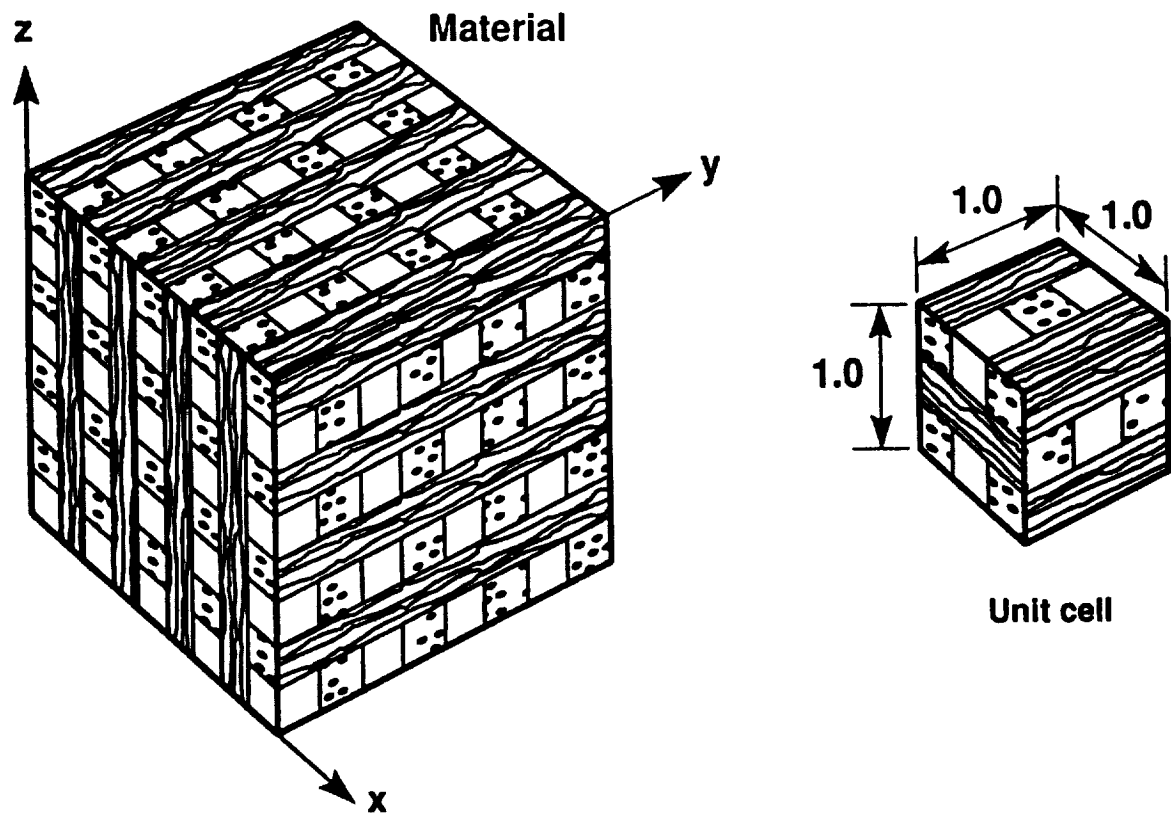


Figure 24. 3-D weave (xyz construction).

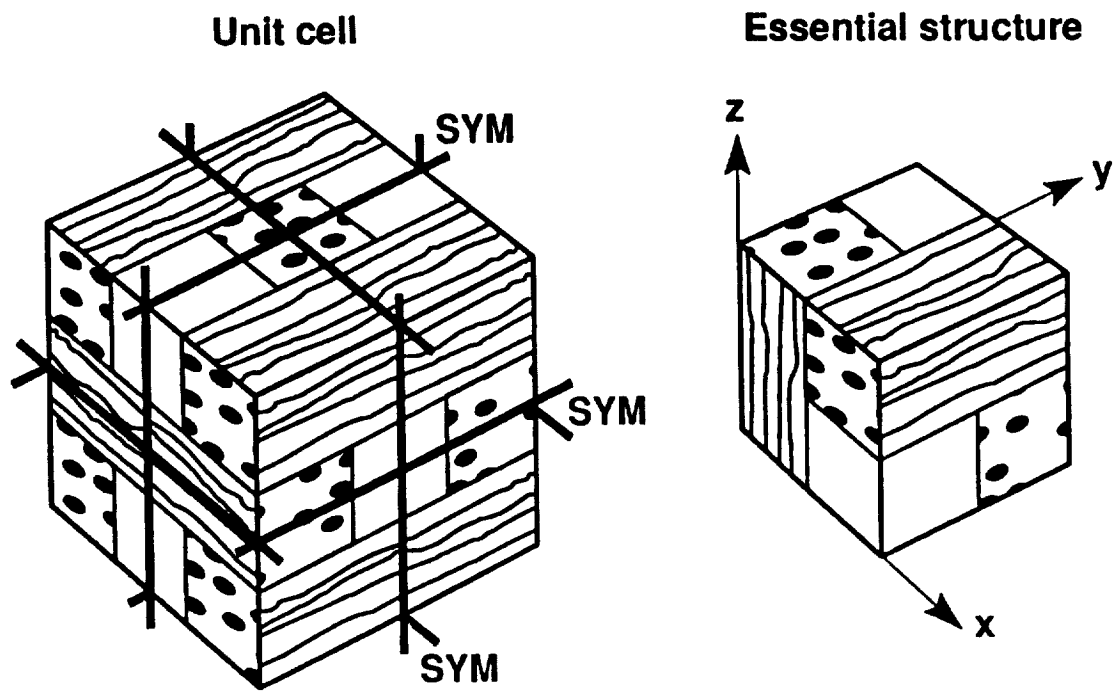


Figure 25. 3-D weave microstructure.

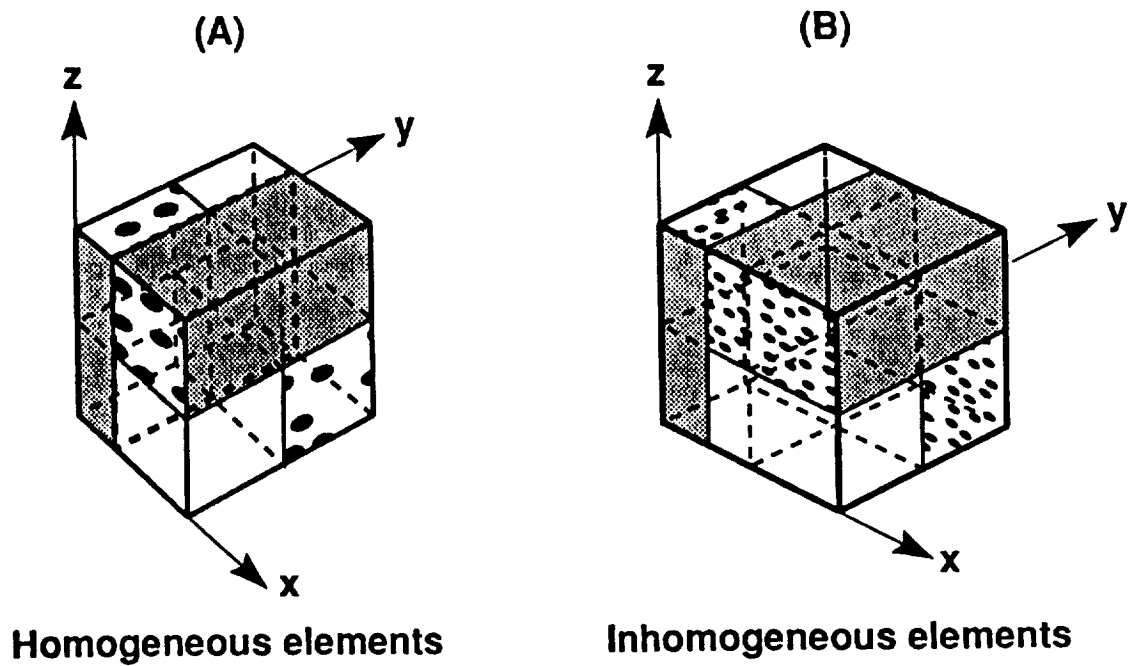


Figure 26. Finite element grids for sample problem #7.

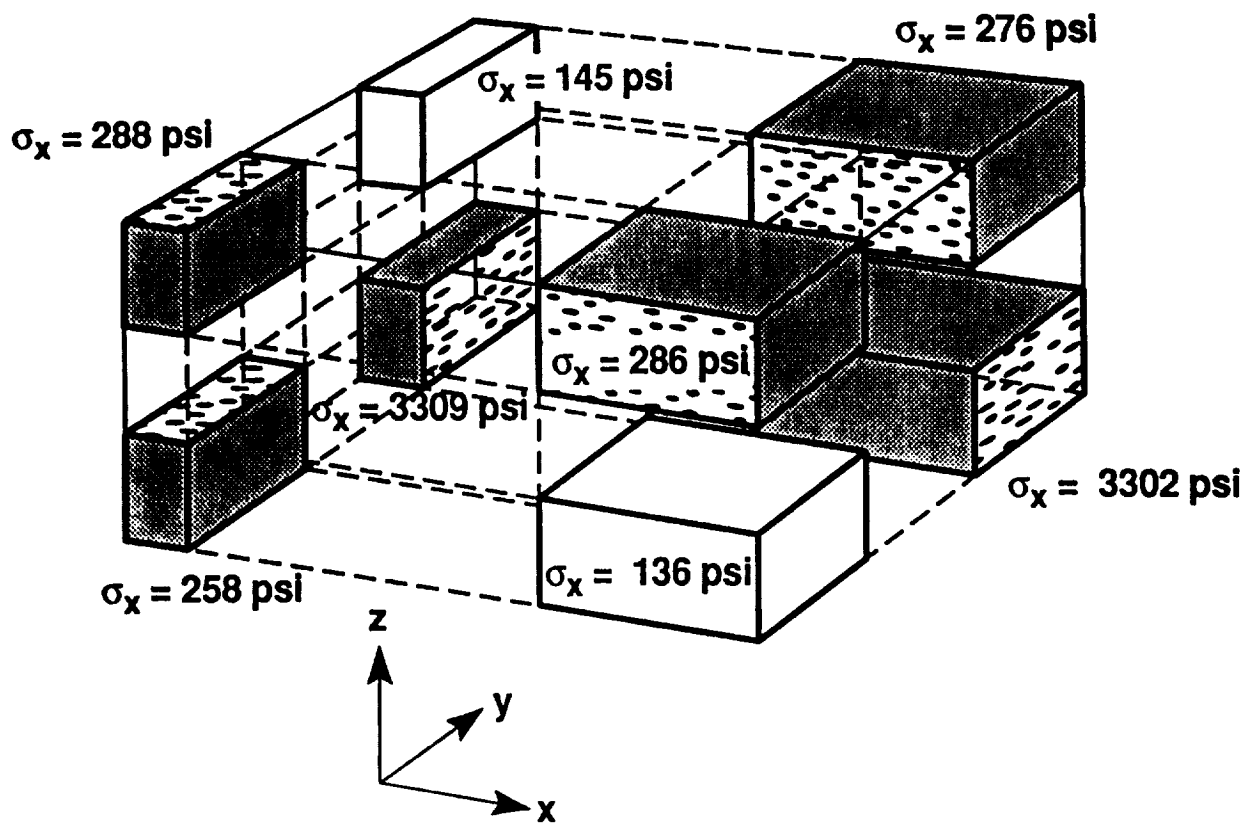


Figure 27. Stresses from homogeneous element analysis.

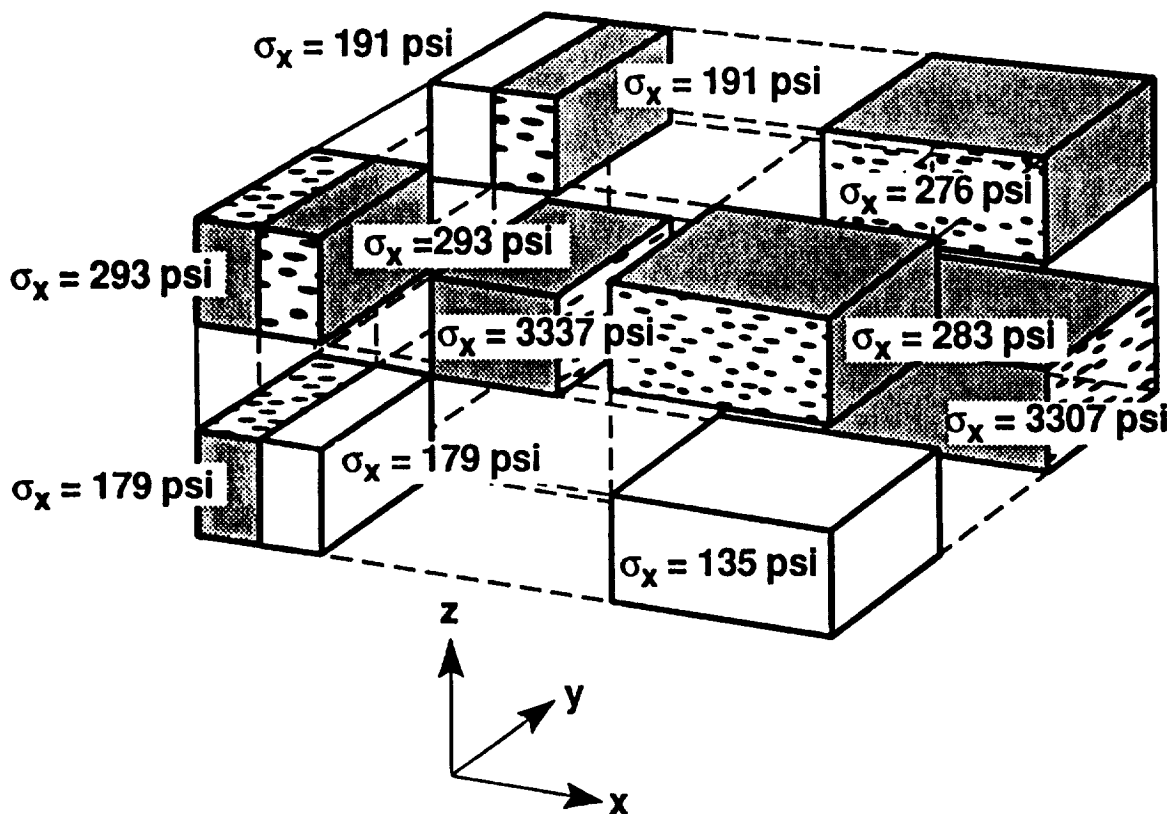


Figure 28. Stresses from inhomogeneous element analysis.

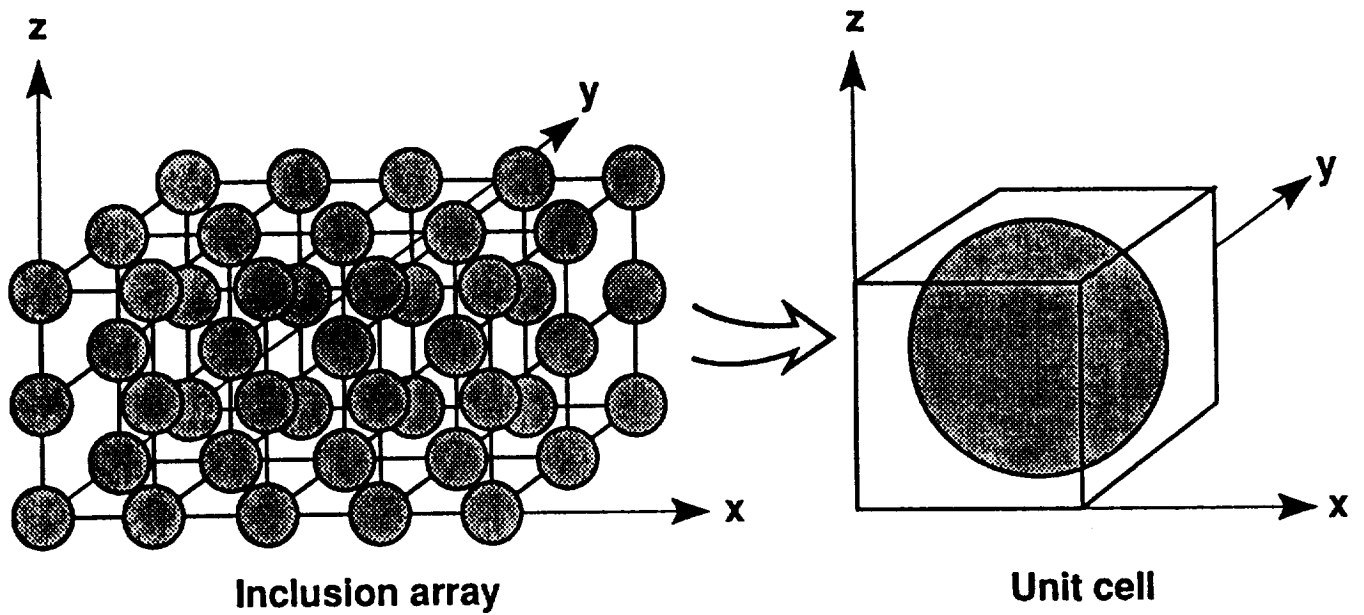
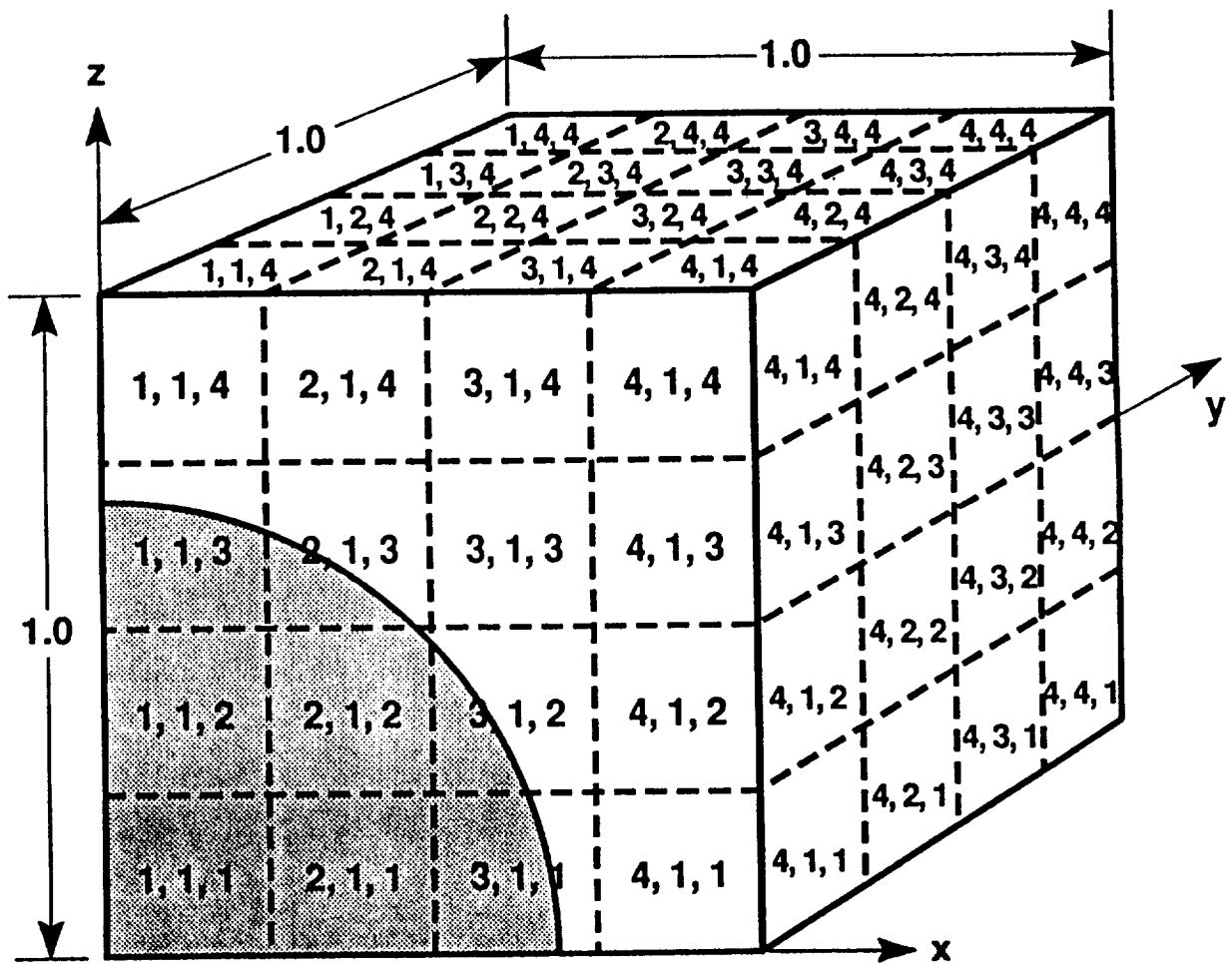


Figure 29. Microgeometry for sample problem #6.



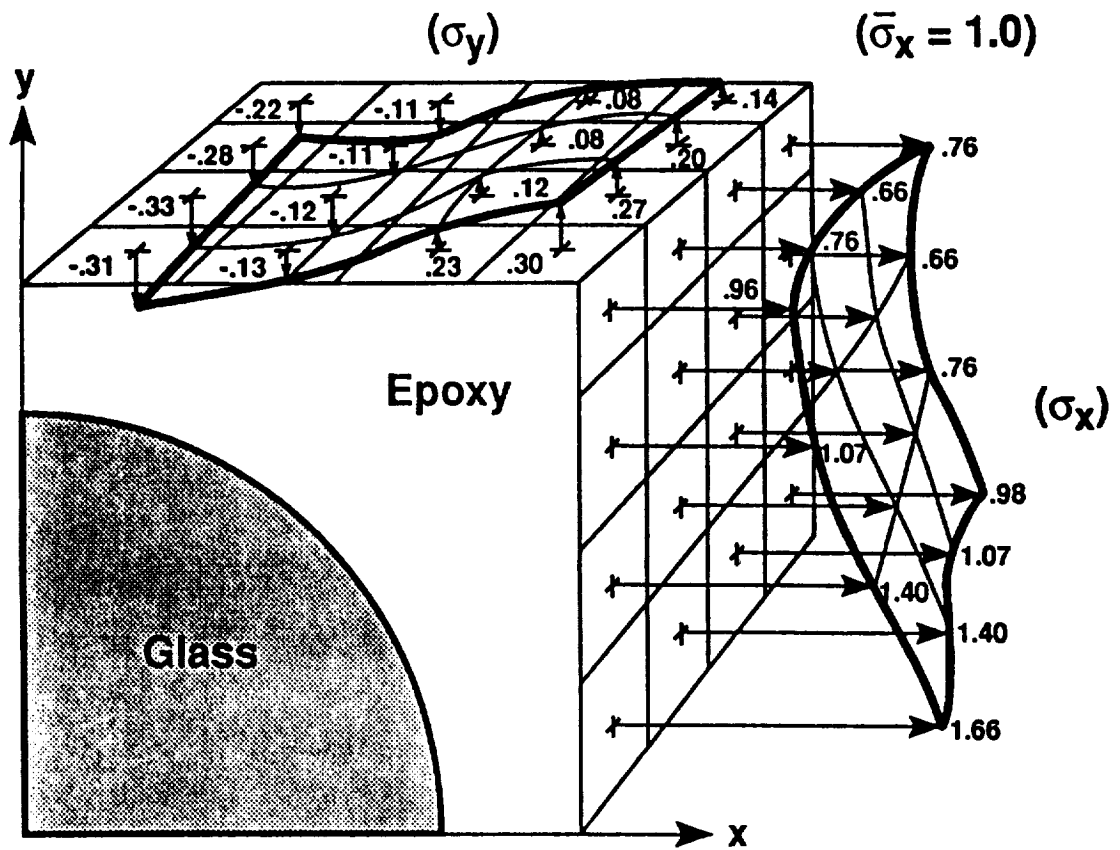


Figure 31. Surface normal stress in sample problem #6.

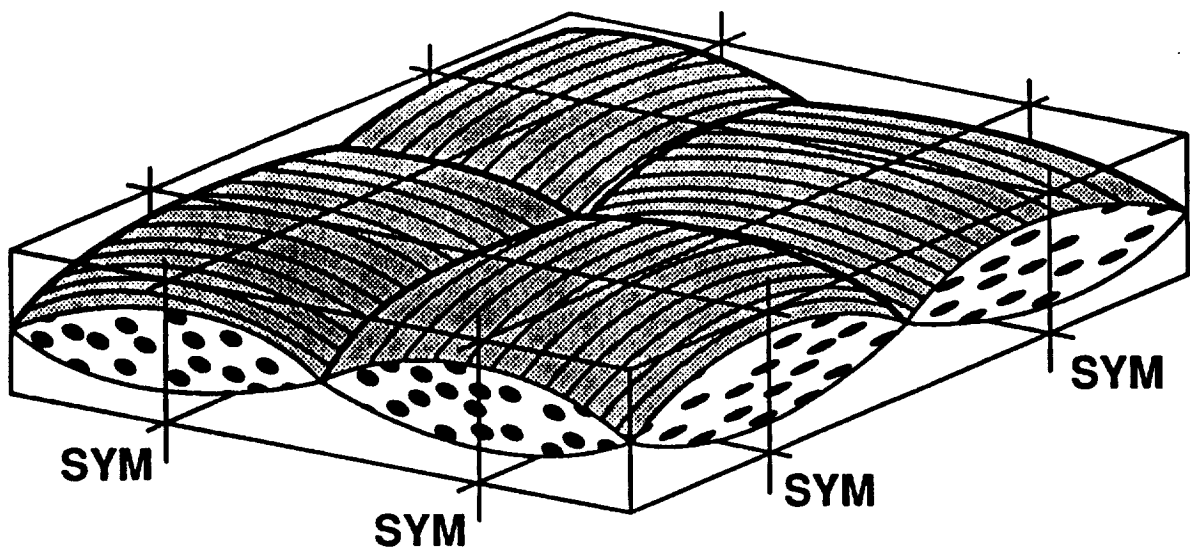


Figure 32. Plain weave unit cell and microgeometry.

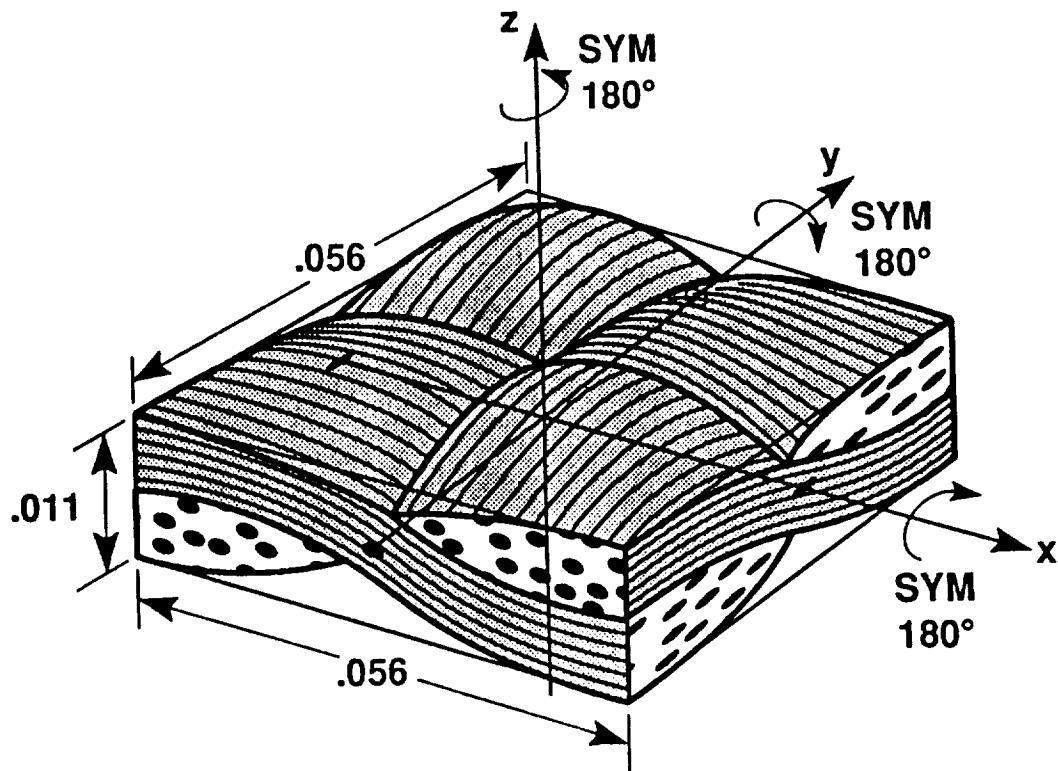


Figure 33. Plain weave reduced volume.

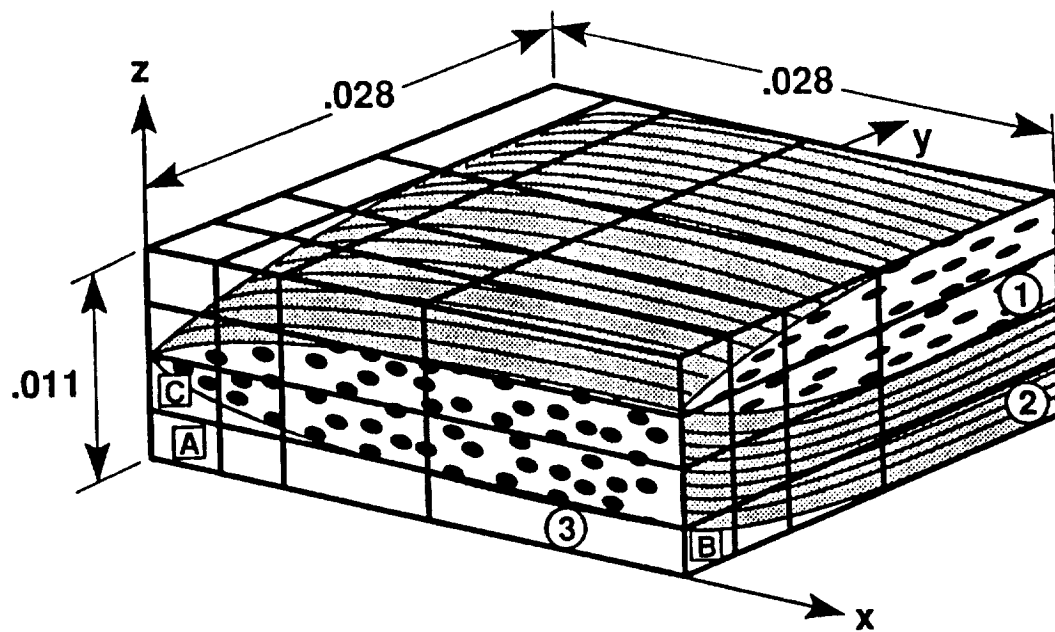


Figure 34. Minimum volume for plain weave analysis.

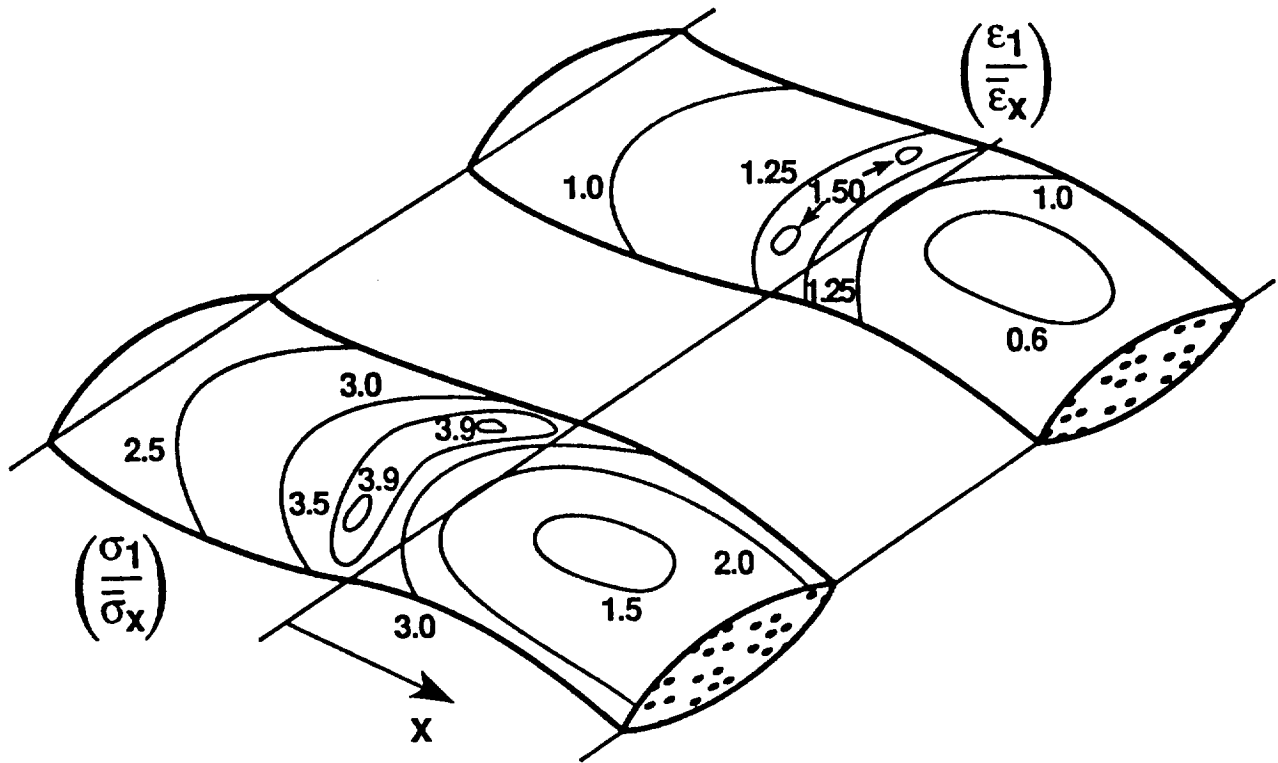


Figure 37. Tow surface stress and strain concentrations.

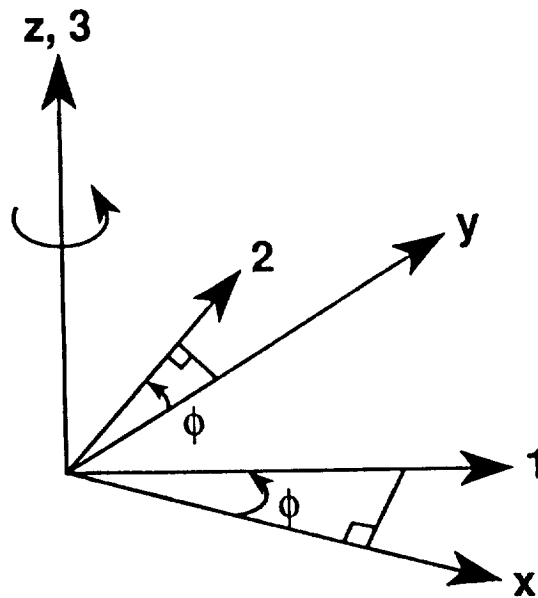


Figure 38. 2-D transformation geometry.

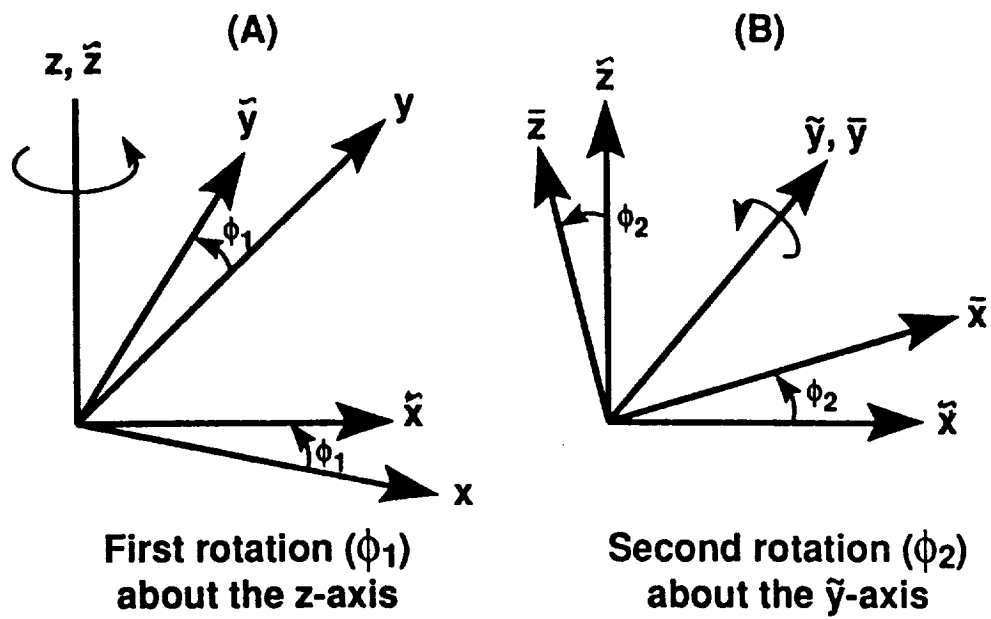


Figure 39. 3-D transformation geometry.

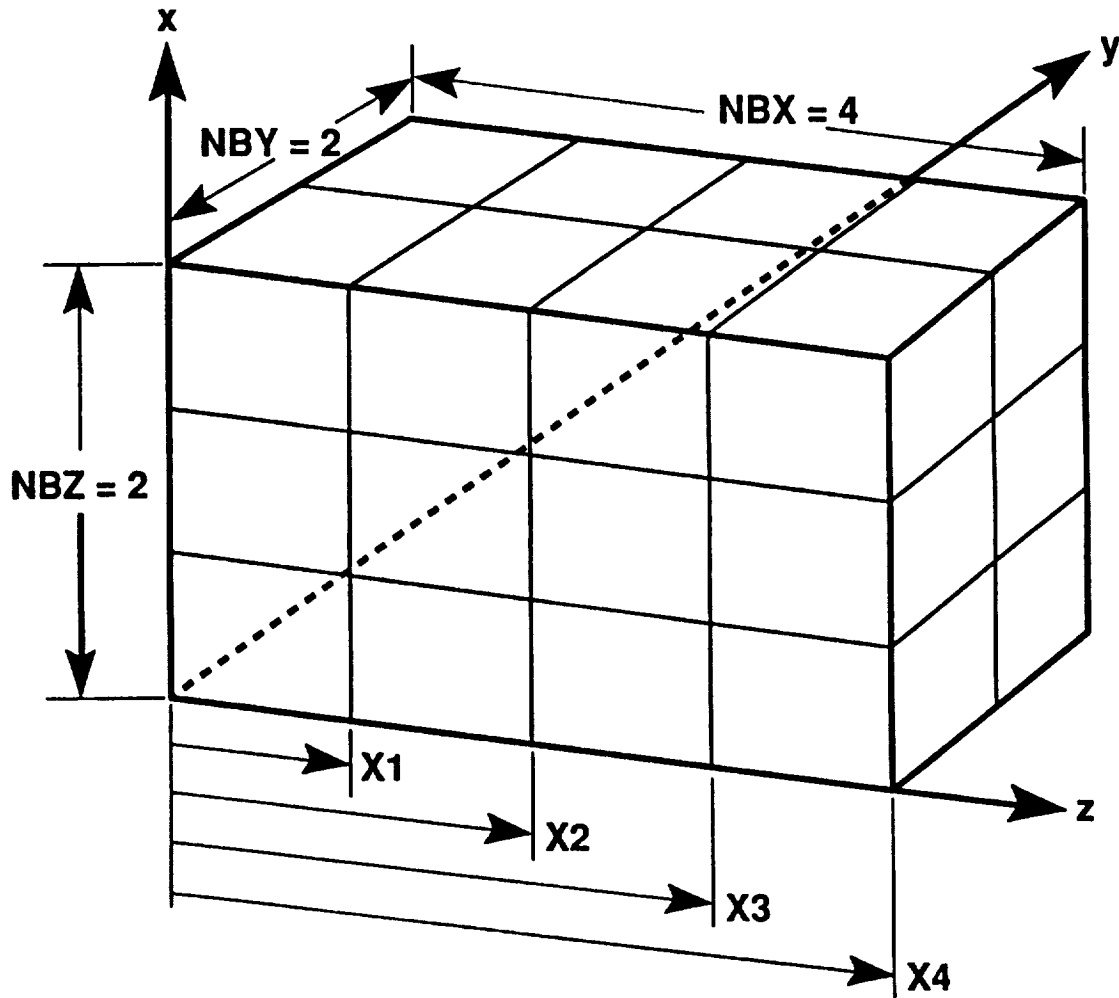


Figure 40. F.E. grid description.

Appendix A

(The 2-D Replacement Element Stress/Strain Relation)

This Appendix considers the replacement of a contiguous volume, filled with two different homogeneous isotropic materials, by a single homogeneous orthotropic material. The original isotropic materials are assumed to be separated by a single flat interfacial plane. It is also assumed that the stresses within each of the original isotropic constituent materials are constant. The pair of dissimilar bulk materials are, in effect, replaced by a series of alternating parallel plates with each layer having the same elastic properties as one of the isotropic materials. The interfaces between each dissimilar plate are parallel to the original bulk constituent interfacial plane. The thicknesses of the alternating layers of each material are in the same proportion as the original volume fractions of bulk constituent materials. As the plate thicknesses diminish the layered structure may be considered to be a composite material in the macroscopic sense. This composite will have a principal axis normal to the interfacial plane and the interfacial plane will be a plane of isotropy of the composite. The following analysis relates to the generalized plane strain response of this composite when the reference plane of the analysis contains the normal to the interfacial plane.

Consider the dissimilar materials i and j, both of which are homogeneous and isotropic with elastic moduli E_i, ν_i, G_i and E_j, ν_j, G_j respectively. The materials are in the form of thin flat sheets bonded together to form a composite as shown in Figure 3. Figure 3 shows two unit cells of the composite. The volume fractions of the two materials are v_i and v_j . Coordinate system 1,2,3 (shown in Figure 3) has the axes 1 and 3 parallel to the material interfacial plane. Axis 2 is normal to that plane. The stress-strain law for material i, for the generalized plane strain case, is given by

$$\begin{Bmatrix} \epsilon_1^i \\ \epsilon_2^i \\ \epsilon_3^i \end{Bmatrix} = \frac{1}{E_i} \begin{bmatrix} 1 & -\nu_i & -\nu_i \\ -\nu_i & 1 & -\nu_i \\ -\nu_i & -\nu_i & 1 \end{bmatrix} \begin{Bmatrix} \sigma_1^i \\ \sigma_2^i \\ \sigma_3^i \end{Bmatrix} \quad \text{-----} \quad (A1)$$

and

$$\gamma_{12}^i = \left(\frac{1}{G_i} \right) \tau_{12}^i . \quad \text{-----} \quad (A2)$$

The corresponding equations for materials j are obtained by substituting j for i in equations (A1) and (A2).

From compatibility of displacements, the composite average strains ($\bar{\epsilon}_1, \bar{\epsilon}_2, \bar{\epsilon}_3, \bar{\gamma}_{12}$) are related to the strains in materials i and j as follows:

$$\left. \begin{aligned} \bar{\epsilon}_1 &= \epsilon_1^i = \epsilon_1^j \\ \bar{\epsilon}_2 &= V_i \epsilon_2^i + V_j \epsilon_2^j \\ \bar{\epsilon}_3 &= \epsilon_3^i = \epsilon_3^j \\ \bar{\gamma}_{12} &= V_i \gamma_{12}^i + V_j \gamma_{12}^j \end{aligned} \right\} \text{--- (A3)}$$

From equilibrium and resolution of forces, the composite stresses ($\bar{\sigma}_1, \bar{\sigma}_2, \bar{\sigma}_3, \bar{\tau}_{12}$) are related to the constituent stresses as follows:

$$\left. \begin{aligned} \bar{\sigma}_1 &= V_i \sigma_1^i + V_j \sigma_1^j \\ \bar{\sigma}_2 &= \sigma_2^i = \sigma_2^j \\ \bar{\sigma}_3 &= V_i \sigma_3^i + V_j \sigma_3^j \\ \bar{\tau}_{12} &= \tau_{12}^i = \tau_{12}^j \end{aligned} \right\} \text{--- (A4)}$$

This system of 20 equations may be solved for the average composite stress/strain relation as follows. First, solve for the constituent stresses using equation (A1) to get

$$\begin{Bmatrix} \sigma_1^i \\ \sigma_2^i \\ \sigma_3^i \end{Bmatrix} = \begin{bmatrix} A^i & B^i & B^i \\ B^i & A^i & B^i \\ B^i & B^i & A^i \end{bmatrix} \begin{Bmatrix} \epsilon_1^i \\ \epsilon_2^i \\ \epsilon_3^i \end{Bmatrix} \text{--- (A5)}$$

where $A^i = E_i(1-\nu_i)/(1+\nu_i)(1-2\nu_i)$ and $B^i = E_i \nu_i/(1+\nu_i)(1-2\nu_i)$.

Since $\sigma_2^i = \sigma_2^j$, it follows that

$$B^i \epsilon_1^i + A^i \epsilon_2^i + B^i \epsilon_3^i = B^j \epsilon_1^j + A^j \epsilon_2^j + B^j \epsilon_3^j.$$

Then, since $\bar{\epsilon}_1 = \epsilon_1^i = \epsilon_1^j$ and $\bar{\epsilon}_3 = \epsilon_3^i = \epsilon_3^j$

$$B^i \bar{\epsilon}_1 + A^i \epsilon_2^i + B^i \bar{\epsilon}_3 = B^j \bar{\epsilon}_1 + A^j \epsilon_2^j + B^j \bar{\epsilon}_3.$$

This equation may be rewritten as

$$(A^i) \epsilon_2^i - (A^j) \epsilon_2^j = (B^j - B^i) \bar{\epsilon}_1 + (B^j - B^i) \bar{\epsilon}_3. \quad \text{————— (A6)}$$

Equations (A3) give another relation between ϵ_2^i and ϵ_2^j , namely,

$$(V_i) \epsilon_2^i + (V_j) \epsilon_2^j = \bar{\epsilon}_2.$$

Solving the two foregoing equations for ϵ_2^i and ϵ_2^j gives

$$\begin{Bmatrix} \epsilon_2^i \\ \epsilon_2^j \end{Bmatrix} = \frac{1}{\Delta} \begin{bmatrix} V_j (B^j - B^i) & A^j & V_j (B^j - B^i) \\ V_i (B^i - B^j) & A^i & V_i (B^i - B^j) \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \bar{\epsilon}_3 \end{Bmatrix} \quad \text{————— (A7)}$$

where $\Delta = A^i V_j + A^j V_i$.

Combining the first three lines from (A4) with (A5) gives

$$\begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_3 \end{Bmatrix} = \sum_{k=i,j} V_k \begin{Bmatrix} \sigma_1^k \\ \sigma_2^k \\ \sigma_3^k \end{Bmatrix} = \sum_{k=i,j} V_k \begin{bmatrix} A^k & B^k & B^k \\ B^k & A^k & B^k \\ B^k & B^k & A^k \end{bmatrix} \begin{Bmatrix} \epsilon_1^k \\ \epsilon_2^k \\ \epsilon_3^k \end{Bmatrix}. \quad \text{————— (A8)}$$

From (A3) and (A7) it follows that

$$\begin{Bmatrix} \epsilon_1^i \\ \epsilon_2^i \\ \epsilon_3^i \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2^i \\ \bar{\epsilon}_3 \end{Bmatrix} \quad \text{————— (A9)}$$

$$\text{and } \epsilon_2^i = [C^i, D^i, C^i] \begin{Bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \bar{\epsilon}_3 \end{Bmatrix} \text{----- (A10)}$$

where $C^i = V_j (B^j - B^i)/\Delta$ and $D^i = A^j/\Delta$.

Combining (A9) and (A10) gives

$$\begin{Bmatrix} \epsilon_1^i \\ \epsilon_2^i \\ \epsilon_3^i \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ C^i & D^i & C^i \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \bar{\epsilon}_3 \end{Bmatrix} \text{----- (A11)}$$

By interchanging i and j

$$\begin{Bmatrix} \epsilon_1^j \\ \epsilon_2^j \\ \epsilon_3^j \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ C^j & D^j & C^j \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \bar{\epsilon}_3 \end{Bmatrix} \text{----- (A12)}$$

where $C^j = V_i (B^i - B^j)/\Delta$ and $D^j = A^i/\Delta$.

Inserting equations (A11) and (A12) into equation (A8) gives

$$\begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_3 \end{Bmatrix} = \left[\sum_{k=i,j} V_k \begin{bmatrix} A^k & B^k & B^k \\ B^k & A^k & B^k \\ B^k & B^k & A^k \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ C^k & D^k & C^k \\ 0 & 0 & 1 \end{bmatrix} \right] \begin{Bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \bar{\epsilon}_3 \end{Bmatrix} \text{----- (A13)}$$

This represents the composite extensional stress/strain relation. The extensional engineering constants for the composite can be obtained from the inverse of the coefficient matrix of equation (A13)

$$\begin{Bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \bar{\epsilon}_3 \end{Bmatrix} = \frac{1}{DET} \begin{bmatrix} (\tilde{A}\tilde{D}-\tilde{B}^2) & \tilde{B}(\tilde{C}-\tilde{A}) & (\tilde{B}^2-\tilde{C}\tilde{D}) \\ \tilde{B}(\tilde{C}-\tilde{A}) & (\tilde{A}^2-\tilde{C}^2) & \tilde{B}(\tilde{C}-\tilde{A}) \\ (\tilde{B}^2-\tilde{C}\tilde{D}) & \tilde{B}(\tilde{C}-\tilde{A}) & (\tilde{A}\tilde{D}-\tilde{B}^2) \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_3 \end{Bmatrix} \quad (A14)$$

where

$$\left. \begin{aligned} \tilde{A} &= V_i(A^i+B^iC^i) + V_j(A^j+B^jC^j) \\ \tilde{B} &= V_i B^i D^i + V_j B^j D^j \\ \tilde{C} &= V_i B^i(C^i+1) + V_j B^j(C^j+1) \\ \tilde{D} &= V_i A^i D^i + V_j A^j D^j \\ DET &= \tilde{A}^2 \tilde{D} - 2\tilde{A}\tilde{B}^2 + 2\tilde{B}^2 \tilde{C} - \tilde{C}^2 \tilde{D} \end{aligned} \right\} \quad (A15)$$

The elastic constants are

$$\left. \begin{aligned} E_1 &= E_3 = DET / (\tilde{A}\tilde{D}-\tilde{B}^2) \\ E_2 &= DET / (\tilde{A}^2-\tilde{C}^2) \\ \frac{\nu_{12}}{E_1} &= \frac{\nu_{21}}{E_2} = \frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3} = \tilde{B}(\tilde{A}-\tilde{C}) / DET \\ \frac{\nu_{13}}{E_1} &= \frac{\nu_{31}}{E_3} = (\tilde{C}\tilde{D}-\tilde{B}^2) / DET \end{aligned} \right\} \quad (A16)$$

The composite shear stress/strain relation is obtained as follows.

Since $\tau_{12}^i = \tau_{12}^j$, $\tau_{12}^i = G^i \gamma_{12}^i$ and $\tau_{12}^j = G^j \gamma_{12}^j$ then

$$(G^i) \gamma_{12}^i - (G^j) \gamma_{12}^j = 0.$$

From (A3)

$$(V_i) \gamma_{12}^i + (V_j) \gamma_{12}^j = \bar{\gamma}_{12}.$$

Solving the two previous equations for γ_{12}^i and γ_{12}^j gives

$$\begin{Bmatrix} \gamma_{12}^i \\ \gamma_{12}^j \end{Bmatrix} = \frac{1}{\nabla} \begin{Bmatrix} G^j \\ G^i \end{Bmatrix} \bar{\gamma}_{12} \quad \text{-----} \quad (A17)$$

where $\nabla = V_i G^j + V_j G^i$.

Since $\bar{\tau}_{12} = \tau_{12}^i = \tau_{12}^j$ then

$$\begin{aligned} \bar{\tau}_{12} &= V_i \tau_{12}^i + V_j \tau_{12}^j \\ &= (V_i G^i) \gamma_{12}^i + (V_j G^j) \gamma_{12}^j \\ &= (V_i G^i) \left(\frac{G^j \bar{\gamma}_{12}}{\nabla} \right) + (V_j G^j) \left(\frac{G^i \bar{\gamma}_{12}}{\nabla} \right) \\ &= \frac{G^i G^j}{\nabla} (V_i + V_j) \bar{\gamma}_{12} \\ &= \left(\frac{G^i G^j}{\nabla} \right) \bar{\gamma}_{12}. \quad \text{-----} \quad (A18) \end{aligned}$$

This is the composite shear stress/strain relation which could also be obtained from the rule of mixtures for stiffnesses in series. Equations (A13) and (A18) can be used to establish a replacement homogeneous stiffness matrix for any such inhomogeneous element. The finite element solution provides the nodal displacements and average strains in each element. Equations (A3), (A7) and (A17) may then be used to obtain the average strains in each constituent material of each element. The constituent stress/strain laws give the constituent stresses. Failure theories may then be applied to each constituent and the interface if desired.

Note that all the foregoing equations apply even when one of the two materials is absent. In this case the stress and strain predictions for the missing material can be ignored.

Appendix B

(Generalized Plane Strain Transformation of Elastic Constants)

The transformation equations for plane stress and plane strain are well known (Ref. 4). However, for the generalized plane strain case, the recognized equations are incomplete. There is another Young's modulus, another coefficient of mutual influence, and two additional Poisson's ratios related to the out of plane (i.e out of the plane of transformation) response. The added Young's modulus normal to the plane of transformation is invariant with respect to the transformation, but the two additional Poisson's ratios and the extra coefficient of mutual influence are not invariants and their transformation equations are not commonly known. This Appendix contains their derivation.

For the coordinate system shown in Figure 26 the stress and strain transformation equations, for generalized plane strain, are given by (Ref. 4),

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 0 & -2sc \\ s^2 & c^2 & 0 & 2sc \\ 0 & 0 & 1 & 0 \\ sc & -sc & 0 & c^2-s^2 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \end{Bmatrix} \quad \text{--- (B1)}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 0 & 2sc \\ s^2 & c^2 & 0 & -2sc \\ 0 & 0 & 1 & 0 \\ -sc & sc & 0 & c^2-s^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix} \quad \text{--- (B2)}$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 0 & -sc \\ s^2 & c^2 & 0 & sc \\ 0 & 0 & 1 & 0 \\ 2sc & -2sc & 0 & c^2-s^2 \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{12} \end{Bmatrix} \quad \text{--- (B3)}$$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} C^2 & S^2 & 0 & SC \\ S^2 & C^2 & 0 & -SC \\ 0 & 0 & 1 & 0 \\ -2SC & 2SC & 0 & C^2-S^2 \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \end{Bmatrix} \quad (B4)$$

where S and C designate $\sin \phi$ and $\cos \phi$ respectively. The stress/strain laws in the principal coordinates of an orthotropic material are (Ref 4)

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 \\ 0 & 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \end{Bmatrix} \quad (B5)$$

In a coordinate system obtained by a rotation of ϕ degrees about the z (or 3) -axis the more general anisotropic equations are apply; namely:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1/E_x & -\nu_{yx}/E_y & -\nu_{zx}/E_z & \eta_{x,xy}/G_{xy} \\ -\nu_{xy}/E_x & 1/E_y & -\nu_{zy}/E_z & \eta_{y,xy}/G_{xy} \\ -\nu_{xz}/E_x & -\nu_{yz}/E_y & 1/E_z & \eta_{z,xy}/G_{xy} \\ \eta_{xy,x}/E_x & \eta_{xy,y}/E_y & \eta_{xy,z}/E_z & 1/G_{xy} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix} \quad (B6)$$

From (B2), (B3) and (B5)

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} C^2 & S^2 & 0 & -SC \\ S^2 & C^2 & 0 & SC \\ 0 & 0 & 1 & 0 \\ 2SC & -2SC & 0 & C^2-S^2 \end{bmatrix} \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} C^2 & S^2 & 0 & 2SC \\ S^2 & C^2 & 0 & -2SC \\ 0 & 0 & 1 & 0 \\ -SC & SC & 0 & C^2-S^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix} \quad (B7)$$

Therefore, from (B6) and (B7) after matrix multiplication

$$\frac{1}{E_x} = \frac{C^4}{E_1} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) S^2 C^2 + \frac{S^4}{E_2}$$

$$\frac{1}{E_y} = \frac{S^4}{E_1} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) S^2 C^2 + \frac{C^4}{E_2}$$

$$* E_z = E_3$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{12}}{E_1} (S^4 + C^4) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}} \right) S^2 C^2$$

$$* \nu_{zx} = \nu_{31} C^2 + \nu_{32} S^2$$

$$* \nu_{zy} = \nu_{31} S^2 + \nu_{32} C^2$$

$$\frac{1}{G_{xy}} = 2 \left(\frac{2}{E_1} + \frac{2}{E_2} + 4 \frac{\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) S^2 C^2 + \frac{1}{G_{12}} (S^4 + C^4)$$

$$\frac{\eta_{xy,x}}{E_x} = \left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) SC^3 - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) S^3 C$$

$$\frac{\eta_{xy,y}}{E_y} = \left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) S^3 C - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) SC^3$$

$$* \eta_{xy,z} = 2 (\nu_{32} - \nu_{31}) SC.$$

These are the complete transformation equations for the generalized plane strain case. The equations preceded by an asterisk are not considered in the plane stress or plane strain case.

The reciprocal relations, from the required symmetry of the stress/strain coefficient matrix, are given by

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y}, \quad \frac{\eta_{x,xy}}{G_{xy}} = \frac{\eta_{xy,x}}{E_x}, \quad \frac{\eta_{y,xy}}{G_{xy}} = \frac{\eta_{xy,y}}{E_y}$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x}, \quad \frac{\nu_{zy}}{E_z} = \frac{\nu_{yz}}{E_y}, \quad \frac{\eta_{z,xy}}{G_{xy}} = \frac{\eta_{xy,z}}{E_z}.$$

The last three of these do not appear in the plane stress (or strain) case.

Appendix C

(The 3-D Replacement Element Stress/Strain Relation)

This Appendix derives the 3-D replacement element stress/strain relation for a subelement containing two elastically dissimilar materials. The two constituent materials are generally anisotropic and separated by an interfacial plane that parallels the \bar{y}, \bar{z} axes of Figure 23. The stresses in any constituent material are assumed to be constant throughout the constituent. From considerations of equilibrium and action/reaction across the interface the following relations exist between the various constituent and average stress components

$$\left. \begin{aligned} \bar{\sigma}_{\bar{x}} &= \sigma_{\bar{x}}^i = \sigma_{\bar{x}}^j \\ \bar{\sigma}_{\bar{y}} &= V_i \sigma_{\bar{y}}^i + V_j \sigma_{\bar{y}}^j \\ \bar{\sigma}_{\bar{z}} &= V_i \sigma_{\bar{z}}^i + V_j \sigma_{\bar{z}}^j \\ \bar{\tau}_{\bar{y}\bar{z}} &= V_i \tau_{\bar{y}\bar{z}}^i + V_j \tau_{\bar{y}\bar{z}}^j \\ \bar{\tau}_{\bar{x}\bar{z}} &= \tau_{\bar{x}\bar{z}}^i = \tau_{\bar{x}\bar{z}}^j \\ \bar{\tau}_{\bar{x}\bar{y}} &= \tau_{\bar{x}\bar{y}}^i = \tau_{\bar{x}\bar{y}}^j \end{aligned} \right\} \quad \text{--- (C1)}$$

From geometric compatibility the following constraint conditions apply to the constituent and average strains.

$$\left. \begin{aligned} \bar{\epsilon}_{\bar{x}} &= V_i \epsilon_{\bar{x}}^i + V_j \epsilon_{\bar{x}}^j \\ \bar{\epsilon}_{\bar{y}} &= \epsilon_{\bar{y}}^i = \epsilon_{\bar{y}}^j \\ \bar{\epsilon}_{\bar{z}} &= \epsilon_{\bar{z}}^i = \epsilon_{\bar{z}}^j \\ \bar{\gamma}_{\bar{y}\bar{z}} &= \gamma_{\bar{y}\bar{z}}^i = \gamma_{\bar{y}\bar{z}}^j \\ \bar{\gamma}_{\bar{x}\bar{z}} &= V_i \gamma_{\bar{x}\bar{z}}^i + V_j \gamma_{\bar{x}\bar{z}}^j \\ \bar{\gamma}_{\bar{x}\bar{y}} &= V_i \gamma_{\bar{x}\bar{y}}^i + V_j \gamma_{\bar{x}\bar{y}}^j \end{aligned} \right\} \quad \text{--- (C2)}$$

Equations (C1) may be rewritten as

$$\left\{ \begin{array}{c} \bar{\sigma}_{\bar{x}} \\ \bar{\sigma}_{\bar{y}} \\ \bar{\sigma}_{\bar{z}} \\ \bar{\tau}_{\bar{y}\bar{z}} \\ \bar{\tau}_{\bar{x}\bar{z}} \\ \bar{\tau}_{\bar{x}\bar{y}} \end{array} \right\} = V_i \left\{ \begin{array}{c} \sigma_{\bar{x}}^i \\ \sigma_{\bar{y}}^i \\ \sigma_{\bar{z}}^i \\ \tau_{\bar{y}\bar{z}}^i \\ \tau_{\bar{x}\bar{z}}^i \\ \tau_{\bar{x}\bar{y}}^i \end{array} \right\} + V_j \left\{ \begin{array}{c} \sigma_{\bar{x}}^j \\ \sigma_{\bar{y}}^j \\ \sigma_{\bar{z}}^j \\ \tau_{\bar{y}\bar{z}}^j \\ \tau_{\bar{x}\bar{z}}^j \\ \tau_{\bar{x}\bar{y}}^j \end{array} \right\} \quad \text{or}$$

$$\begin{Bmatrix} \bar{\sigma}_{\bar{x}} \\ \bar{\sigma}_{\bar{y}} \\ \bar{\sigma}_{\bar{z}} \\ \bar{\tau}_{\bar{y}\bar{z}} \\ \bar{\tau}_{\bar{x}\bar{z}} \\ \bar{\tau}_{\bar{x}\bar{y}} \end{Bmatrix} = v_i [C^i] \begin{Bmatrix} \bar{\epsilon}_{\bar{x}}^i \\ \bar{\epsilon}_{\bar{y}}^i \\ \bar{\epsilon}_{\bar{z}}^i \\ \bar{\gamma}_{\bar{y}\bar{z}}^i \\ \bar{\gamma}_{\bar{x}\bar{z}}^i \\ \bar{\gamma}_{\bar{x}\bar{y}}^i \end{Bmatrix} + v_j [C^j] \begin{Bmatrix} \bar{\epsilon}_{\bar{x}}^j \\ \bar{\epsilon}_{\bar{y}}^j \\ \bar{\epsilon}_{\bar{z}}^j \\ \bar{\gamma}_{\bar{y}\bar{z}}^j \\ \bar{\gamma}_{\bar{x}\bar{z}}^j \\ \bar{\gamma}_{\bar{x}\bar{y}}^j \end{Bmatrix} \quad (C3)$$

where the matrices $[C^i]$ and $[C^j]$ are the coefficient matrices of the constituent material stress/strain equations in the $\bar{x}, \bar{y}, \bar{z}$ coordinate system of Figure 23. From equations (C1), (C2) and (C3)

$$\begin{bmatrix} C_{11}^i & C_{15}^i & C_{16}^i - C_{11}^j - C_{15}^j - C_{16}^j \\ v_i & 0 & 0 & v_j & 0 & 0 \\ C_{51}^i & C_{55}^i & C_{56}^i - C_{51}^j - C_{55}^j - C_{56}^j \\ 0 & v_i & 0 & 0 & v_j & 0 \\ C_{61}^i & C_{65}^i & C_{66}^i - C_{61}^j - C_{65}^j - C_{66}^j \\ 0 & 0 & v_i & 0 & 0 & v_j \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_{\bar{x}}^i \\ \bar{\gamma}_{\bar{x}\bar{z}}^i \\ \bar{\gamma}_{\bar{x}\bar{y}}^i \\ \bar{\epsilon}_{\bar{x}}^j \\ \bar{\gamma}_{\bar{x}\bar{z}}^j \\ \bar{\gamma}_{\bar{x}\bar{y}}^j \end{Bmatrix} =$$

$$\begin{bmatrix} 0 & -\bar{C}_{12} - \bar{C}_{13} - \bar{C}_{14} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\bar{C}_{52} - \bar{C}_{53} - \bar{C}_{54} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\bar{C}_{62} - \bar{C}_{63} - \bar{C}_{64} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_{\bar{x}} \\ \bar{\epsilon}_{\bar{y}} \\ \bar{\epsilon}_{\bar{z}} \\ \bar{\gamma}_{\bar{y}\bar{z}} \\ \bar{\gamma}_{\bar{x}\bar{z}} \\ \bar{\gamma}_{\bar{y}\bar{x}} \end{Bmatrix} \quad (C4)$$

where \bar{C}_{mn} designates $(C_{mn}^i - C_{mn}^j)$ and C_{mn}^i is the mn th element of $[C^i]$. If $[\bar{D}]$ is the inverse (obtained numerically) of the leading coefficient matrix in equation (C4) then

$$\begin{Bmatrix} \epsilon_{\bar{x}}^i \\ \gamma_{\bar{x}\bar{z}}^i \\ \gamma_{\bar{x}\bar{y}}^i \\ \epsilon_{\bar{x}}^j \\ \gamma_{\bar{x}\bar{z}}^j \\ \gamma_{\bar{x}\bar{y}}^j \end{Bmatrix} = \begin{bmatrix} \bar{D}_{11} & \bar{D}_{12} & \bar{D}_{13} & \bar{D}_{14} & \bar{D}_{15} & \bar{D}_{16} \\ \bar{D}_{21} & \bar{D}_{22} & \bar{D}_{23} & \bar{D}_{24} & \bar{D}_{25} & \bar{D}_{26} \\ \bar{D}_{31} & \bar{D}_{32} & \bar{D}_{33} & \bar{D}_{34} & \bar{D}_{35} & \bar{D}_{36} \\ \bar{D}_{41} & \bar{D}_{42} & \bar{D}_{43} & \bar{D}_{44} & \bar{D}_{45} & \bar{D}_{46} \\ \bar{D}_{51} & \bar{D}_{52} & \bar{D}_{53} & \bar{D}_{54} & \bar{D}_{55} & \bar{D}_{56} \\ \bar{D}_{61} & \bar{D}_{62} & \bar{D}_{63} & \bar{D}_{64} & \bar{D}_{65} & \bar{D}_{66} \end{bmatrix} \begin{bmatrix} 0 & -\bar{C}_{12} & -\bar{C}_{13} & -\bar{C}_{14} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\bar{C}_{52} & -\bar{C}_{53} & -\bar{C}_{54} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\bar{C}_{62} & -\bar{C}_{63} & -\bar{C}_{64} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_{\bar{x}} \\ \bar{\epsilon}_{\bar{y}} \\ \bar{\epsilon}_{\bar{z}} \\ \bar{\gamma}_{\bar{y}\bar{z}} \\ \bar{\gamma}_{\bar{x}\bar{z}} \\ \bar{\gamma}_{\bar{x}\bar{y}} \end{Bmatrix} \quad (C5)$$

where \bar{D}_{mn} is the mn th element of $[\bar{D}]$. Substituting values of the constituent strains from (C5) into (C3) gives the following stress/strain law for the replacement material within the element of Figure 23.

$$\begin{Bmatrix} \bar{\sigma}_{\bar{x}} \\ \bar{\sigma}_{\bar{y}} \\ \bar{\sigma}_{\bar{z}} \\ \bar{\tau}_{\bar{y}\bar{z}} \\ \bar{\tau}_{\bar{x}\bar{z}} \\ \bar{\tau}_{\bar{x}\bar{y}} \end{Bmatrix} = [\bar{A}] \begin{Bmatrix} \bar{\epsilon}_{\bar{x}} \\ \bar{\epsilon}_{\bar{y}} \\ \bar{\epsilon}_{\bar{z}} \\ \bar{\gamma}_{\bar{y}\bar{z}} \\ \bar{\gamma}_{\bar{x}\bar{z}} \\ \bar{\gamma}_{\bar{x}\bar{y}} \end{Bmatrix} \quad (C6)$$

where

$$\bar{A}_{11} = V_i (C_{11}^i \bar{D}_{12} + C_{15}^i \bar{D}_{22} + C_{16}^i \bar{D}_{32}) + V_j (C_{11}^j \bar{D}_{42} + C_{15}^j \bar{D}_{52} + C_{16}^j \bar{D}_{62})$$

$$\bar{A}_{12} = V_i (C_{21}^i \bar{D}_{12} + C_{25}^i \bar{D}_{22} + C_{26}^i \bar{D}_{32}) + V_j (C_{21}^j \bar{D}_{42} + C_{25}^j \bar{D}_{52} + C_{26}^j \bar{D}_{62})$$

$$\bar{A}_{13} = V_i (C_{31}^i \bar{D}_{12} + C_{35}^i \bar{D}_{22} + C_{36}^i \bar{D}_{32}) + V_j (C_{31}^j \bar{D}_{42} + C_{35}^j \bar{D}_{52} + C_{36}^j \bar{D}_{62})$$

$$\bar{A}_{14} = V_i (C_{41}^i \bar{D}_{12} + C_{45}^i \bar{D}_{22} + C_{46}^i \bar{D}_{32}) + V_j (C_{41}^j \bar{D}_{42} + C_{45}^j \bar{D}_{52} + C_{46}^j \bar{D}_{62})$$

$$\bar{A}_{15} = V_i (C_{51}^i \bar{D}_{12} + C_{55}^i \bar{D}_{22} + C_{56}^i \bar{D}_{32}) + V_j (C_{51}^j \bar{D}_{42} + C_{55}^j \bar{D}_{52} + C_{56}^j \bar{D}_{62})$$

$$\bar{A}_{16} = V_i (C_{61}^i \bar{D}_{12} + C_{65}^i \bar{D}_{22} + C_{66}^i \bar{D}_{32}) + V_j (C_{61}^j \bar{D}_{42} + C_{65}^j \bar{D}_{52} + C_{66}^j \bar{D}_{62})$$

$$\begin{aligned} \bar{A}_{22} = & V_i \left\{ C_{22}^i + C_{21}^i (\bar{D}_{11} \bar{C}_{21} + \bar{D}_{13} \bar{C}_{25} + \bar{D}_{15} \bar{C}_{26}) \right. \\ & + C_{25}^i (\bar{D}_{21} \bar{C}_{21} + \bar{D}_{23} \bar{C}_{25} + \bar{D}_{25} \bar{C}_{26}) \\ & \left. + C_{26}^i (\bar{D}_{31} \bar{C}_{21} + \bar{D}_{33} \bar{C}_{25} + \bar{D}_{35} \bar{C}_{26}) \right\} \\ & + V_j \left\{ C_{22}^j + C_{21}^j (\bar{D}_{41} \bar{C}_{21} + \bar{D}_{43} \bar{C}_{25} + \bar{D}_{45} \bar{C}_{26}) \right. \\ & + C_{25}^j (\bar{D}_{51} \bar{C}_{21} + \bar{D}_{53} \bar{C}_{25} + \bar{D}_{55} \bar{C}_{26}) \\ & \left. + C_{26}^j (\bar{D}_{61} \bar{C}_{21} + \bar{D}_{63} \bar{C}_{25} + \bar{D}_{65} \bar{C}_{26}) \right\} \end{aligned}$$

$$\begin{aligned} \bar{A}_{23} = & V_i \left\{ C_{23}^i + C_{21}^i (\bar{D}_{11} \bar{C}_{31} + \bar{D}_{13} \bar{C}_{35} + \bar{D}_{15} \bar{C}_{36}) \right. \\ & + C_{25}^i (\bar{D}_{21} \bar{C}_{31} + \bar{D}_{23} \bar{C}_{35} + \bar{D}_{25} \bar{C}_{36}) \\ & \left. + C_{26}^i (\bar{D}_{31} \bar{C}_{31} + \bar{D}_{33} \bar{C}_{35} + \bar{D}_{35} \bar{C}_{36}) \right\} \\ & + V_j \left\{ C_{23}^j + C_{21}^j (\bar{D}_{41} \bar{C}_{31} + \bar{D}_{43} \bar{C}_{35} + \bar{D}_{45} \bar{C}_{36}) \right. \\ & + C_{25}^j (\bar{D}_{51} \bar{C}_{31} + \bar{D}_{53} \bar{C}_{35} + \bar{D}_{55} \bar{C}_{36}) \\ & \left. + C_{26}^j (\bar{D}_{61} \bar{C}_{31} + \bar{D}_{63} \bar{C}_{35} + \bar{D}_{65} \bar{C}_{36}) \right\} \end{aligned}$$

$$\begin{aligned} \bar{A}_{24} = & V_i \left\{ C_{24}^i + C_{21}^i (\bar{D}_{11} \bar{C}_{41} + \bar{D}_{13} \bar{C}_{45} + \bar{D}_{15} \bar{C}_{46}) \right. \\ & + C_{25}^i (\bar{D}_{21} \bar{C}_{41} + \bar{D}_{23} \bar{C}_{45} + \bar{D}_{25} \bar{C}_{46}) \\ & \left. + C_{26}^i (\bar{D}_{31} \bar{C}_{41} + \bar{D}_{33} \bar{C}_{45} + \bar{D}_{35} \bar{C}_{46}) \right\} \end{aligned}$$

$$+ V_j \left\{ C_{24}^j + C_{21}^j (\bar{D}_{41} \bar{C}_{41} + \bar{D}_{43} \bar{C}_{45} + \bar{D}_{45} \bar{C}_{46}) \right. \\
+ C_{25}^j (\bar{D}_{51} \bar{C}_{41} + \bar{D}_{53} \bar{C}_{45} + \bar{D}_{55} \bar{C}_{46}) \\
\left. + C_{26}^j (\bar{D}_{61} \bar{C}_{41} + \bar{D}_{63} \bar{C}_{45} + \bar{D}_{65} \bar{C}_{46}) \right\}$$

$$\bar{A}_{25} = V_i (C_{21}^i \bar{D}_{14} + C_{25}^i \bar{D}_{24} + C_{26}^i \bar{D}_{34}) + V_j (C_{21}^j \bar{D}_{44} + C_{25}^j \bar{D}_{54} + C_{26}^j \bar{D}_{64})$$

$$\bar{A}_{26} = V_i (C_{21}^i \bar{D}_{16} + C_{25}^i \bar{D}_{26} + C_{26}^i \bar{D}_{36}) + V_j (C_{21}^j \bar{D}_{46} + C_{25}^j \bar{D}_{56} + C_{26}^j \bar{D}_{66})$$

$$\bar{A}_{33} = V_i \left\{ C_{33}^i + C_{31}^i (\bar{D}_{11} \bar{C}_{31} + \bar{D}_{13} \bar{C}_{35} + \bar{D}_{15} \bar{C}_{36}) \right. \\
+ C_{35}^i (\bar{D}_{21} \bar{C}_{31} + \bar{D}_{23} \bar{C}_{35} + \bar{D}_{25} \bar{C}_{36}) \\
+ C_{36}^i (\bar{D}_{31} \bar{C}_{31} + \bar{D}_{33} \bar{C}_{35} + \bar{D}_{35} \bar{C}_{36}) \left. \right\} \\
+ V_j \left\{ C_{33}^j + C_{31}^j (\bar{D}_{41} \bar{C}_{31} + \bar{D}_{43} \bar{C}_{35} + \bar{D}_{45} \bar{C}_{36}) \right. \\
+ C_{35}^j (\bar{D}_{51} \bar{C}_{31} + \bar{D}_{53} \bar{C}_{35} + \bar{D}_{55} \bar{C}_{36}) \\
\left. + C_{36}^j (\bar{D}_{61} \bar{C}_{31} + \bar{D}_{63} \bar{C}_{35} + \bar{D}_{65} \bar{C}_{36}) \right\}$$

$$\bar{A}_{34} = V_i \left\{ C_{34}^i + C_{31}^i (\bar{D}_{11} \bar{C}_{41} + \bar{D}_{13} \bar{C}_{45} + \bar{D}_{15} \bar{C}_{46}) \right. \\
+ C_{35}^i (\bar{D}_{21} \bar{C}_{41} + \bar{D}_{23} \bar{C}_{45} + \bar{D}_{25} \bar{C}_{46}) \\
+ C_{36}^i (\bar{D}_{31} \bar{C}_{41} + \bar{D}_{33} \bar{C}_{45} + \bar{D}_{35} \bar{C}_{46}) \left. \right\} \\
+ V_j \left\{ C_{34}^j + C_{31}^j (\bar{D}_{41} \bar{C}_{41} + \bar{D}_{43} \bar{C}_{45} + \bar{D}_{45} \bar{C}_{46}) \right. \\
+ C_{35}^j (\bar{D}_{51} \bar{C}_{41} + \bar{D}_{53} \bar{C}_{45} + \bar{D}_{55} \bar{C}_{46}) \\
\left. + C_{36}^j (\bar{D}_{61} \bar{C}_{41} + \bar{D}_{63} \bar{C}_{45} + \bar{D}_{65} \bar{C}_{46}) \right\}$$

$$\bar{A}_{35} = V_i (C_{31}^i \bar{D}_{14} + C_{35}^i \bar{D}_{24} + C_{36}^i \bar{D}_{34}) + V_j (C_{31}^j \bar{D}_{44} + C_{35}^j \bar{D}_{54} + C_{36}^j \bar{D}_{64})$$

$$\bar{A}_{36} = V_i (C_{31}^i \bar{D}_{16} + C_{35}^i \bar{D}_{26} + C_{36}^i \bar{D}_{36}) + V_j (C_{31}^j \bar{D}_{46} + C_{35}^j \bar{D}_{56} + C_{36}^j \bar{D}_{66})$$

$$\begin{aligned}\bar{A}_{44} = & V_i \left\{ C_{44}^i + C_{41}^i (\bar{D}_{11} \bar{C}_{41} + \bar{D}_{13} \bar{C}_{45} + \bar{D}_{15} \bar{C}_{46}) \right. \\ & + C_{45}^i (\bar{D}_{21} \bar{C}_{41} + \bar{D}_{23} \bar{C}_{45} + \bar{D}_{25} \bar{C}_{46}) \\ & \left. + C_{46}^i (\bar{D}_{31} \bar{C}_{41} + \bar{D}_{33} \bar{C}_{45} + \bar{D}_{35} \bar{C}_{46}) \right\} \\ & + V_j \left\{ C_{44}^j + C_{41}^j (\bar{D}_{41} \bar{C}_{41} + \bar{D}_{45} \bar{C}_{45} + \bar{D}_{45} \bar{C}_{46}) \right. \\ & + C_{45}^j (\bar{D}_{51} \bar{C}_{41} + \bar{D}_{53} \bar{C}_{45} + \bar{D}_{55} \bar{C}_{46}) \\ & \left. + C_{46}^j (\bar{D}_{61} \bar{C}_{41} + \bar{D}_{63} \bar{C}_{45} + \bar{D}_{65} \bar{C}_{46}) \right\}\end{aligned}$$

$$\bar{A}_{45} = V_i (C_{41}^i \bar{D}_{14} + C_{45}^i \bar{D}_{24} + C_{46}^i \bar{D}_{34}) + V_j (C_{41}^j \bar{D}_{44} + C_{45}^j \bar{D}_{54} + C_{46}^j \bar{D}_{64})$$

$$\bar{A}_{46} = V_i (C_{41}^i \bar{D}_{16} + C_{45}^i \bar{D}_{26} + C_{46}^i \bar{D}_{36}) + V_j (C_{41}^j \bar{D}_{46} + C_{45}^j \bar{D}_{56} + C_{46}^j \bar{D}_{66})$$

$$\bar{A}_{55} = V_i (C_{51}^i \bar{D}_{14} + C_{55}^i \bar{D}_{24} + C_{56}^i \bar{D}_{34}) + V_j (C_{51}^j \bar{D}_{44} + C_{55}^j \bar{D}_{54} + C_{56}^j \bar{D}_{64})$$

$$\bar{A}_{56} = V_i (C_{51}^i \bar{D}_{16} + C_{55}^i \bar{D}_{26} + C_{56}^i \bar{D}_{36}) + V_j (C_{51}^j \bar{D}_{46} + C_{55}^j \bar{D}_{56} + C_{56}^j \bar{D}_{66})$$

$$\bar{A}_{66} = V_i (C_{61}^i \bar{D}_{16} + C_{65}^i \bar{D}_{26} + C_{66}^i \bar{D}_{36}) + V_j (C_{61}^j \bar{D}_{46} + C_{65}^j \bar{D}_{56} + C_{66}^j \bar{D}_{66}).$$

The matrix $[\bar{A}]$ is symmetric.

The replacement stress/strain equation (C6) must be transformed into the global x,y,z coordinate system for use in forming the stiffness matrix of the replacement element. When the finite element solution to the deformations of the unit cell is obtained the average strains in each element can be computed. In a replacement element the average strains can be transformed into the $\bar{x}, \bar{y}, \bar{z}$ interfacial coordinate system. Equations (C2) and (C3) will then give the average strains in each constituent material. These strains can be transformed into the principal axis of the material and the constituent stresses computed. Any yield or initial failure criteria can then be applied to the constituent materials or the interface.

Appendix D

(The 3-D Stress/Strain Transformations)

Any 3-D stress/strain coefficient matrix in the $\tilde{x}, \tilde{y}, \tilde{z}$ coordinate system of Figure 39A can be transformed into the global x, y, z coordinate system in two stages. The first stage consists of a rotation of ϕ_1 degrees about the z -coordinate axis of Figure 39A. The positive sense of rotation for ϕ_1 is clockwise to an observer at the origin looking in the positive z -direction. From the equilibrium of triangular wedge elements whose faces are normal to the coordinate axes it can be shown that the stresses in both coordinates are related as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c_1^2 & s_1^2 & 0 & 0 & 0 & -2s_1c_1 \\ s_1^2 & c_1^2 & 0 & 0 & 0 & 2s_1c_1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & s_1 & 0 \\ 0 & 0 & 0 & -s_1 & c_1 & 0 \\ s_1c_1 & -s_1c_1 & 0 & 0 & 0 & (c_1^2 - s_1^2) \end{bmatrix} \begin{Bmatrix} \sigma_{\tilde{x}} \\ \sigma_{\tilde{y}} \\ \sigma_{\tilde{z}} \\ \tau_{\tilde{y}\tilde{z}} \\ \tau_{\tilde{x}\tilde{z}} \\ \tau_{\tilde{x}\tilde{y}} \end{Bmatrix} \quad \text{--- (D1)}$$

where $S_1 = \sin \phi_1$ and $C_1 = \cos \phi_1$. The reverse or inverse transformation is readily obtained by substituting $-\phi_1$ in place of ϕ_1 in the previous equations giving

$$\begin{Bmatrix} \sigma_{\tilde{x}} \\ \sigma_{\tilde{y}} \\ \sigma_{\tilde{z}} \\ \tau_{\tilde{y}\tilde{z}} \\ \tau_{\tilde{x}\tilde{z}} \\ \tau_{\tilde{x}\tilde{y}} \end{Bmatrix} = \begin{bmatrix} c_1^2 & s_1^2 & 0 & 0 & 0 & 2s_1c_1 \\ s_1^2 & c_1^2 & 0 & 0 & 0 & -2s_1c_1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & -s_1 & 0 \\ 0 & 0 & 0 & s_1 & c_1 & 0 \\ -s_1c_1 & s_1c_1 & 0 & 0 & 0 & (c_1^2 - s_1^2) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \quad \text{--- (D2)}$$

By definition the two coefficient matrices in (D1) and (D2) must be the inverses of each other. The strain transformation equations for the same rotation of ϕ_1 degree about the z-coordinate are derived from geometric considerations alone and given by

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} c_1^2 & s_1^2 & 0 & 0 & 0 & -s_1 c_1 \\ s_1^2 & c_1^2 & 0 & 0 & 0 & s_1 c_1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & s_1 & 0 \\ 0 & 0 & 0 & -s_1 & c_1 & 0 \\ 2s_1 c_1 & -2s_1 c_1 & 0 & 0 & 0 & (c_1^2 - s_1^2) \end{bmatrix} \begin{Bmatrix} \epsilon_{\bar{x}} \\ \epsilon_{\bar{y}} \\ \epsilon_{\bar{z}} \\ \gamma_{\bar{y}\bar{z}} \\ \gamma_{\bar{x}\bar{z}} \\ \gamma_{\bar{x}\bar{y}} \end{Bmatrix} \quad \text{--- (D3)}$$

with the reverse transformation

$$\begin{Bmatrix} \epsilon_{\bar{x}} \\ \epsilon_{\bar{y}} \\ \epsilon_{\bar{z}} \\ \gamma_{\bar{y}\bar{z}} \\ \gamma_{\bar{x}\bar{z}} \\ \gamma_{\bar{x}\bar{y}} \end{Bmatrix} = \begin{bmatrix} c_1^2 & s_1^2 & 0 & 0 & 0 & s_1 c_1 \\ s_1^2 & c_1^2 & 0 & 0 & 0 & -s_1 c_1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & -s_1 & 0 \\ 0 & 0 & 0 & s_1 & c_1 & 0 \\ -2s_1 c_1 & 2s_1 c_1 & 0 & 0 & 0 & (c_1^2 - s_1^2) \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \quad \text{--- (D4)}$$

The second stage of the transformation consists of a rotation of ϕ_2 degrees about the \bar{y} -coordinate as shown in Figure 39B. In this case the positive sense of rotation for ϕ_2 is counter clockwise to an observer at the origin looking in the positive \bar{y} -direction. The relationships between the stresses and strains before and after this latter transformation are

$$\begin{pmatrix} \sigma_{\bar{x}} \\ \sigma_{\bar{y}} \\ \sigma_{\bar{z}} \\ T_{\bar{y}\bar{z}} \\ T_{\bar{x}\bar{z}} \\ T_{\bar{x}\bar{y}} \end{pmatrix} = \begin{bmatrix} c_2^2 & 0 & s_2^2 & 0 & -2s_2c_2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ s_2^2 & 0 & c_2^2 & 0 & 2s_2c_2 & 0 \\ 0 & 0 & 0 & c_2 & 0 & s_2 \\ s_2c_2 & 0 & -s_2c_2 & 0 & (c_2^2 - s_2^2) & 0 \\ 0 & 0 & 0 & -s_2 & 0 & c_2 \end{bmatrix} \begin{pmatrix} \sigma_{\bar{x}} \\ \sigma_{\bar{y}} \\ \sigma_{\bar{z}} \\ T_{\bar{y}\bar{z}} \\ T_{\bar{x}\bar{z}} \\ T_{\bar{x}\bar{y}} \end{pmatrix} \quad (D5)$$

$$\begin{pmatrix} \sigma_{\bar{x}} \\ \sigma_{\bar{y}} \\ \sigma_{\bar{z}} \\ T_{\bar{y}\bar{z}} \\ T_{\bar{x}\bar{z}} \\ T_{\bar{x}\bar{y}} \end{pmatrix} = \begin{bmatrix} c_2^2 & 0 & s_2^2 & 0 & 2s_2c_2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ s_2^2 & 0 & c_2^2 & 0 & -2s_2c_2 & 0 \\ 0 & 0 & 0 & c_2 & 0 & -s_2 \\ -s_2c_2 & 0 & s_2c_2 & 0 & (c_2^2 - s_2^2) & 0 \\ 0 & 0 & 0 & s_2 & 0 & c_2 \end{bmatrix} \begin{pmatrix} \sigma_{\bar{x}} \\ \sigma_{\bar{y}} \\ \sigma_{\bar{z}} \\ T_{\bar{y}\bar{z}} \\ T_{\bar{x}\bar{z}} \\ T_{\bar{x}\bar{y}} \end{pmatrix} \quad (D6)$$

$$\begin{pmatrix} \epsilon_{\bar{x}} \\ \epsilon_{\bar{y}} \\ \epsilon_{\bar{z}} \\ \gamma_{\bar{y}\bar{z}} \\ \gamma_{\bar{x}\bar{z}} \\ \gamma_{\bar{x}\bar{y}} \end{pmatrix} = \begin{bmatrix} c_2^2 & 0 & s_2^2 & 0 & -s_2c_2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ s_2^2 & 0 & c_2^2 & 0 & s_2c_2 & 0 \\ 0 & 0 & 0 & c_2 & 0 & s_2 \\ 2s_2c_2 & 0 & -2s_2c_2 & 0 & (c_2^2 - s_2^2) & 0 \\ 0 & 0 & 0 & -s_2 & 0 & c_2 \end{bmatrix} \begin{pmatrix} \epsilon_{\bar{x}} \\ \epsilon_{\bar{y}} \\ \epsilon_{\bar{z}} \\ \gamma_{\bar{y}\bar{z}} \\ \gamma_{\bar{x}\bar{z}} \\ \gamma_{\bar{x}\bar{y}} \end{pmatrix} \quad (D7)$$

$$\begin{pmatrix} \epsilon_{\bar{x}} \\ \epsilon_{\bar{y}} \\ \epsilon_{\bar{z}} \\ \gamma_{\bar{y}\bar{z}} \\ \gamma_{\bar{x}\bar{z}} \\ \gamma_{\bar{x}\bar{y}} \end{pmatrix} = \begin{bmatrix} c_2^2 & 0 & s_2^2 & 0 & s_2c_2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ s_2^2 & 0 & c_2^2 & 0 & -s_2c_2 & 0 \\ 0 & 0 & 0 & c_2 & 0 & -s_2 \\ -2s_2c_2 & 0 & 2s_2c_2 & 0 & (c_2^2 - s_2^2) & 0 \\ 0 & 0 & 0 & s_2 & 0 & c_2 \end{bmatrix} \begin{pmatrix} \epsilon_{\bar{x}} \\ \epsilon_{\bar{y}} \\ \epsilon_{\bar{z}} \\ \gamma_{\bar{y}\bar{z}} \\ \gamma_{\bar{x}\bar{z}} \\ \gamma_{\bar{x}\bar{y}} \end{pmatrix} \quad (D8)$$

where $S_2 = \sin \phi_2$ and $C_2 = \cos \phi_2$.

The stresses and strains in the final $\bar{x}, \bar{y}, \bar{z}$ -coordinates are thus related to the original stresses and strains in the x, y, z coordinates by the following equations obtained by multiplying the appropriate pair of coefficient matrices in equations (D3) through (D8) in the correct order.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_1^2 C_2^2 & S_1^2 & C_1^2 S_2^2 & 2S_1 C_1 S_2 & -2C_1^2 S_2 C_2 & -2S_1 C_1 C_2 \\ S_1^2 C_2^2 & C_1^2 & S_1^2 S_2^2 & -2S_1 C_1 S_2 & -2S_1^2 S_2 C_2 & 2S_1 C_1 C_2 \\ S_2^2 & 0 & C_2^2 & 0 & 2S_2 C_2 & 0 \\ S_1 S_2 C_2 & 0 & -S_1 S_2 C_2 & C_1 C_2 & S_1 (C_2^2 - S_2^2) & C_1 S_2 \\ C_1 S_2 C_2 & 0 & -C_1 S_2 C_2 & -S_1 C_2 & C_1 (C_2^2 - S_2^2) & -S_1 S_2 \\ S_1 C_1 C_2^2 & -S_1 C_1 & S_1 C_1 S_2^2 & -(C_1^2 - S_1^2) S_2 & -2S_1 C_1 S_2 C_2 & (C_1^2 - S_1^2) C_2 \end{bmatrix} \begin{Bmatrix} \sigma_{\bar{x}} \\ \sigma_{\bar{y}} \\ \sigma_{\bar{z}} \\ \tau_{\bar{y}\bar{z}} \\ \tau_{\bar{x}\bar{z}} \\ \tau_{\bar{x}\bar{y}} \end{Bmatrix} \quad \text{--- (D9)}$$

$$\begin{Bmatrix} \sigma_{\bar{x}} \\ \sigma_{\bar{y}} \\ \sigma_{\bar{z}} \\ \tau_{\bar{y}\bar{z}} \\ \tau_{\bar{x}\bar{z}} \\ \tau_{\bar{x}\bar{y}} \end{Bmatrix} = \begin{bmatrix} C_1^2 C_2^2 & S_1^2 C_1^2 & S_2^2 & 2S_1 S_2 C_2 & 2C_1 S_2 C_2 & 2S_1 C_1 C_2^2 \\ S_1^2 & C_1^2 & 0 & 0 & 0 & -2S_1 C_1 \\ C_1^2 S_2^2 & S_1^2 S_2^2 & C_2^2 & -2S_1 S_2 C_2 & -2C_1 S_2 C_2 & 2S_1 C_1 S_2^2 \\ S_1 C_1 S_2 & -S_1 C_1 S_2 & 0 & C_1 C_2 & -S_1 C_2 & -(C_1^2 - S_1^2) S_2 \\ -C_1^2 S_2 C_2 & -S_1^2 S_2 C_2 & S_2 C_2 & S_1 (C_2^2 - S_2^2) & C_1 (C_2^2 - S_2^2) & -2S_1 C_1 S_2 C_2 \\ -S_1 C_1 C_2 & S_1 C_1 C_2 & 0 & C_1 S_2 & -S_1 S_2 & (C_1^2 - S_1^2) C_2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \quad \text{--- (D10)}$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} C_1^2 C_2^2 & S_1^2 & C_1^2 S_2^2 & S_1 C_1 S_2 & -C_1^2 S_2 C_2 & -S_1 C_1 C_2 \\ S_1^2 C_2^2 & C_1^2 & S_1^2 S_2^2 & -S_1 C_1 S_2 & -S_1^2 S_2 C_2 & S_1 C_1 C_2 \\ S_2^2 & 0 & C_2^2 & 0 & S_2 C_2 & 0 \\ 2S_1 S_2 C_2 & 0 & -2S_1 S_2 C_2 & C_1 C_2 & S_1 (C_2^2 - S_2^2) & C_1 S_2 \\ 2C_1 S_2 C_2 & 0 & -2C_1 S_2 C_2 & -S_1 C_2 & C_1 (C_2^2 - S_2^2) & -S_1 S_2 \\ 2S_1 S_2 C_2^2 - 2S_1 C_1 & 2S_1 C_1 S_2^2 & -(C_1^2 - S_1^2) S_2 & -2S_1 C_1 S_2 C_2 & (C_1^2 - S_1^2) C_2 & \end{bmatrix} \begin{Bmatrix} \epsilon_{\bar{x}} \\ \epsilon_{\bar{y}} \\ \epsilon_{\bar{z}} \\ \gamma_{\bar{y}\bar{z}} \\ \gamma_{\bar{x}\bar{z}} \\ \gamma_{\bar{x}\bar{y}} \end{Bmatrix} \quad \text{--- (D11)}$$

$$\begin{Bmatrix} \epsilon_{\tilde{x}} \\ \epsilon_{\tilde{y}} \\ \epsilon_{\tilde{z}} \\ \gamma_{\tilde{y}\tilde{z}} \\ \gamma_{\tilde{x}\tilde{z}} \\ \gamma_{\tilde{x}\tilde{y}} \end{Bmatrix} = \begin{bmatrix} C_1^2 C_2^2 & S_1^2 C_2^2 & S_2^2 & S_1 S_2 C_2 & C_1 S_2 C_2 & S_1 C_1 C_2^2 \\ S_1^2 & C_1^2 & 0 & 0 & 0 & -S_1 C_1 \\ C_1^2 S_2^2 & S_1^2 S_2^2 & C_2^2 & -S_1 S_2 C_2 & -C_1 S_2 C_2 & S_1 C_1 S_2^2 \\ 2S_1 C_1 S_2 & -2S_1 C_1 S_2 & 0 & C_1 C_2 & -S_1 C_2 & -(C_1^2 - S_1^2) S_2 \\ -2C_1^2 S_2 C_2 & -2S_1^2 S_2 C_2 & 2S_2 C_2 & S_1 (C_2^2 - S_2^2) & C_1 (C_2^2 - S_2^2) & -2S_1 C_1 S_2 C_2 \\ -2S_1 C_1 C_2 & 2S_1 C_1 C_2 & 0 & C_1 S_2 & -S_1 S_2 & (C_1^2 - S_1^2) C_2 \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \quad \text{--- (D12)}$$

Note that the coefficient matrix in (D11) is the transpose of the coefficient matrix in (D10) and the coefficient matrices in (D9) and (D12) are also transposes of each other. This fact considerably simplifies the use of these equations.

Appendix E

(Instructions for Use of the 3-D Computer Program for Resolving Stresses in a Unit Cell)

The program REPLACE is a FORTRAN code that analyzes a 3-D unit cell structure for the composite moduli and the internal stresses in each material of each element of the finite element model of the unit cell. The only element in the program is the eight-node isoparametric brick element. The stiffness matrix for this element is formed by numerical integration over eight Gaussian integration points. The boundary conditions of structural symmetry are assumed to apply on all faces of the unit cell.

The input data can be considered to be four different data packages. The first package serves to establish an ad hoc file of constituent materials and their elastic properties for use in the current run. The second data package establishes the hexagonal finite element grid for the unit cell. The third data package contains the constituent material distribution information for each brick element in the finite element grid. The last data package contains the average applied stresses to be used for the detailed internal stress computations. Each data package will be described in the foregoing sequence, starting with the constituent materials file.

The program operates interactively and is largely self-explanatory through the prompt messages. The first block of input data serves to establish an ad hoc array of constituent materials and their elastic constants for use in the current run. These constituents can be selected from the eight sets of resident materials whose properties correspond to unidirectional high, medium and low modulus graphite/epoxy, unidirectional glass/epoxy, bulk aluminum, bulk epoxy, etc. (see Table 5). New sets of material properties can be input either by inserting new DATA statements at the beginning of the program or by following a sequence of material input prompts.

The first piece of input data is a single digit integer (NM) that specifies the number of constituent materials to appear in the ad hoc list of materials for that run. Not all of the materials on the list need to be used and the same material may appear more than once. Each material is presumed to be orthotropic with a plane of isotropy. Thus, six elastic constants suffice to define the linear response of the material. Whatever, additional material constants are associated with the yield or failure criteria must also be input. The six elastic constants (in the sequence in which they are input and stored) are the Young's modulus in the principal reinforcing direction, the Young's modulus in the plane of isotropy, the longitudinal/transverse Poisson's ratio, the Poisson's ratio in the plane of isotropy, the longitudinal/transverse shear modulus, and the shear modulus in the plane of isotropy. A maximum strain failure criteria is currently in the program. It requires four additional constants per material. The four input constants associated with this criteria are the

longitudinal tension and compression strain and the transverse tension and compression strain. The initial input number of materials (NM) establishes a do loop for filling the material property (MP) array that contains the ten input constants for each of the (NM) constituent materials. The program requests a constituent material data location number for data insertion in the first row of the MP array. If a material number between one and eight is specified then the ten material constants from that data statement are inserted in the first row of the MP array for the properties of what will be known subsequently as constituent material number one. If a data statement number greater than eight is called for then the program makes ten queries for each material property to be inserted in the first row of the MP array. This sequence is repeated until each row of the material property array is filled. The constituent material input sequence also establishes a numbering scheme for recalling the constituent material properties. The first row of MP array to be filled is henceforth material number one, the second is material number two, etc.

Following the material property selection is the description of the geometry of the rectangular finite element grid to be used in the analysis of the unit cell structure. The first input quantity specifies the side length of the unit cell in the x-direction. The second quantity is an integer (NBX) that specifies the number of brick elements along one side of the unit cell in the x-direction. If NBX is greater than one then the distance of each node point from the origin in the x-direction must be specified. This is done by specifying the x-distance from the origin to the farthest interior point of each element that lies along the x-axis of Figure 40, starting with the nearest element to the origin and ending with the farthest one. Each distance is designated as a percentage of the x-side length of the unit cell. The last distance in the x-direction is not specified, but is assumed to be 100% of the unit cell side length in that direction. The same set of quantities are then specified for the y-direction of the unit cell grid and then repeated once more for the z-direction. This establishes the finite element grid.

It remains to describe the material distribution within each brick element. This is done by means of a triple nested do loop starting with the brick element closest to the origin of Figure 40. The material distribution within that element is described in its entirety before the inner do loop indexes to the next element in the plus z-direction for the same material distribution data. This inner do-loop continues to index in the z-direction until the last element touching the z-axis is fully described. The middle do-loop then indexes to the second brick element (from the origin) along the y-axis. The inner do-loop once again ranges over each element in this second stack of brick elements, requisitioning materials information for each in the same sequence. When each z-stacking of brick elements along the $x=0$ face of the unit cell is described then the outer do-loop indexes to the next brick element along the x-axis and the two inner do-loops are restarted. Now consider the way the distribution of material in each brick element is described.

The material distribution within a brick element must be reducible to a series of constituent material junctions as shown in Figure 35A. Each branch or trunk represents a different material with the two branch materials combined to form the trunk material (as described in Appendix C). The main trunk of the tree structure represents the single material to be used in that element stiffness matrix calculation.

The tree structure can, in principal, be as complex as necessary as long as each junction contains no more than two branches and one trunk. However, in practice, each finite element has its tree structure limited to two junctions. No more complexity was required for the example problems considered. With this limitation a finite element can not contain more than three different constituent materials, as shown in Figure 35C. Each of the three outer branches must contain a single constituent material chosen from one of the sets of material properties established in the MP array. The description of the outer branch consists of the material designation number, corresponding to the MP row number, and the pair of spherical coordinate angles (ϕ_1, ϕ_2) that specify the grain or fiber direction, as shown in Figure 39. Each junction must also contain a description of the volume fraction of each branch and the pair of spherical coordinate angles (ψ_1, ψ_2) that specify the direction of the normal to the interfacial plane separating the two branch materials (see Figure 23). Before inputting any unit cell analysis problem the tree structure of each finite element should be sketched and labeled as shown in Figure 35.

In summary, fifteen data values are needed to describe the most general two-junction material arrangement in each element. These numbers are prompted and input in the following sequence (with reference to Figure 35A):

- a) material property (MP) array row designation for branch ①
- b) spherical angle ϕ_1 for fiber direction in branch ①
- c) spherical angle ϕ_2 for fiber direction in branch ①
- d) material property (MP) array row designation in branch ②
- e) spherical angle ϕ_1 for fiber direction in branch ②
- f) spherical angle ϕ_2 for fiber direction in branch ②
- g) volume fraction of branch ① material at junction ①
- h) spherical angle ψ_1 , for interfacial normal at junction ①
- i) spherical angle ψ_2 for interfacial normal at junction ①
- j) material property (MP) array row designation for branch ③

- k) spherical angle ϕ_1 for fiber direction in branch ③
- l) spherical angle ϕ_2 for fiber direction in branch ③
- m) volume fraction of branch ③ material at junction ⑧
- n) spherical angle ψ_1 for interfacial normal at junction ⑧
- o) spherical angle ψ_2 for interfacial normal at junction ⑧

If there is only one junction in an element (or no junctions) less input information is needed. For one junction only the first nine inputs ((a)-(i)) are needed. For no junctions only the first three inputs ((a)-(c)) are needed. To trigger the correct set of prompts, the first piece of input data for any element is the integer 0 through 2 that specifies the number of junctions in the tree structure. Then the appropriate set of prompts will automatically follow in the foregoing sequence. All angles are to be specified in degrees and decimal fractions of a degree. Volume fractions of materials are specified in decimal fractions form (0.0 to 1.0.) This completes the description of the material content of each element. The stress output for each element is given in the reverse order of the sequence of junction descriptors. The stresses are given in the principal axes of the constituent material. Each material will also have its minimum margin of safety computed based on a maximum strain criteria (with respect to the principal axes of the material).

The only remaining input is the specification of the six components of the 3-D, applied, far-field stresses for which the internal stresses in each material in each element are to be computed.

The 3-D weave (or XYZ material) serves as a simple example for controlling and responding to the unit cell analysis program input prompts. The composite consists of three sets of unidirectional graphite/epoxy tows interspersed as shown in Figure 24. The tows in the x,y and z direction are all of a different size. The x-direction tow fills 25% of the unit cell. The y-direction tow fills 37.5% of the unit cell. The z-direction tow fills 12.5% of the unit cell. The remaining 25% of the volume is bulk epoxy. The finite element mesh could easily be adjusted so that each element was homogeneous. However, for illustrative purposes the mesh will be set up such that there are three inhomogeneous elements (out of a total of eight) each containing two different constituent materials. The grid is chosen such that all the finite elements have the same dimensions, and there are two elements stacked in each coordinate direction. There are only two materials needed: unidirectional graphite/epoxy (the first material among the DATA statements) and bulk epoxy (the sixth material in the DATA statements). Thus, the first three input integers

declare that two materials are needed and that they are constituent materials number one and six. Since material one was listed first it becomes material number one for the rest of the run. Material six hereafter becomes material number two. A printout of the series of program prompts and responses are given at the end of this Appendix along with the stiffness and stress output. The input is echoed in double parentheses to distinguish it from the prompts. The unit cell dimensions are 2.0 units in the x,y and z-directions. The center node point along each edge of the unit cell divides the edge into two equal lengths. The exploded sketch of Figure 28 shows the sequence in which the eight elements are described. Elements ① , ② and ④ are inhomogeneous.

The first element contains both constituent materials: unidirectional graphite/epoxy and bulk epoxy, in equal volumes. The fiber direction angles are $\phi_1=0^\circ$ and $\phi_2=90^\circ$ for material ① . The volume fraction for material one is 0.5. Any direction angles can be specified for material two, the isotropic bulk epoxy. In this case $\phi_1=0^\circ$ and $\phi_2=0^\circ$ were specified. The interfacial normal has $\psi_1 = \psi_2 = 0^\circ$ as its direction. This information fully describes the material content of the first element.

Element two contains only one constituent material, but half of the fibers are going in the y-direction and half are going in the z-direction. This can be represented by a single junction with both branches made from constituent material number one. One branch has fiber direction angles of $\phi_1 = 0^\circ$ and $\phi_2 = 90^\circ$. The other branch has fiber direction angles of $\phi_1 = 90^\circ$ and $\phi_2 = 0^\circ$. The volume fraction of branch ① material is 0.5 and the interfacial normal has the direction angles $\psi_1 = 0^\circ$ and $\psi_2 = 0^\circ$.

The third element is homogeneous in material one with no junctions. The fiber directions are $\phi_1=\phi_2=0^\circ$.

The fourth element differs from the first only in the fiber direction angles. The rest of the elements are homogeneous with no junctions.

The average applied stress is 1000 psi tension in the x-direction with the other stress components equal to zero.

The output consists of the composite moduli and the stresses in the principal axes of each constituent material in each element. Minimum margins of safety are also given.

INPUT NO. COMPOSITE MATERIALS NEEDED,NM
 ((2))
 SELECT A MATERIAL NUMBER FROM ONE TO TEN
 ((1))
 SELECT A MATERIAL NUMBER FROM ONE TO TEN
 ((6))

MATERIAL PROPERTY DATA ECHO
 18000000.00 1500000.00 0.23 0.30 700000.00 700000.00
 0.1000E-01 0.1000E-01 0.1000E-01 0.1000E-01
 500000.00 500000.00 0.35 0.35 180000.00 180000.00
 0.1000E-01 0.1000E-01 0.1000E-01 0.1000E-01

INPUT SIDE LENGTH OF UNIT CELL IN X DIR.
 ((2.0))
 INPUT NO. SUBCELLS (X DIR.) IN UNIT CELL
 ((2))
 INPUT DIST.(%) ORIGIN TO UNIT CELL NODE 2
 ((50.0))

INPUT SIDE LENGTH OF UNIT CELL IN Y DIR.
 ((2.0))
 INPUT NO. SUBCELLS (Y DIR.) IN UNIT CELL
 ((2))
 INPUT DIST.(%) ORIGIN TO UNIT CELL NODE 2
 ((50.0))

INPUT SIDE LENGTH OF UNIT CELL IN Z DIR.
 ((2.0))
 INPUT NO. SUBCELLS (Z DIR.) IN UNIT CELL
 ((2))
 INPUT DIST.(%) ORIGIN TO UNIT CELL NODE 2
 ((50.0))

INPUT NUMBER OF JUNCTIONS AT LOCATION 1 1 1
 ((1))
 INPUT MATL. NO. 1 AT 1 1 1
 ((1))
 INPUT 1ST FIBER SPHERICAL ANGLE
 ((0.0))
 INPUT 2ND FIBER SPHERICAL ANGLE
 ((90.0))
 INPUT MATL. NO. 2 AT 1 1 1
 ((2))
 INPUT 1ST FIBER SPHERICAL ANGLE
 ((0.0))
 INPUT 2ND FIBER SPHERICAL ANGLE
 ((0.0))
 INPUT 1ST MATL. VOLUME FRACTION
 ((0.5))
 INPUT 1ST INTERFACIAL NORMAL ANGLE
 ((0.0))
 INPUT 2ND INTERFACIAL NORMAL ANGLE
 ((0.0))

INPUT NUMBER OF JUNCTIONS AT LOCATION 1 1 2
 ((1))
 INPUT MATL. NO. 1 AT 1 1 2
 ((1))
 INPUT 1ST FIBER SPHERICAL ANGLE
 ((0.0))
 INPUT 2ND FIBER SPHERICAL ANGLE
 ((90.0))
 INPUT MATL. NO. 2 AT 1 1 2
 ((1))
 INPUT 1ST FIBER SPHERICAL ANGLE

```

      ((90.0))
INPUT 2ND FIBER SPHERICAL ANGLE
      (( 0.0))
INPUT 1ST MATL. VOLUME FRACTION
      (( 0.5))
INPUT 1ST INTERFACIAL NORMAL ANGLE
      (( 0.0))
INPUT 2ND INTERFACIAL NORMAL ANGLE
      (( 0.0))

INPUT NUMBER OF JUNCTIONS AT LOCATION   1   2   1
      (( 0 ))
SPECIFY THE CURRENT MATL. ID. NO.
      (( 1 ))
INPUT 1ST FIBER SPHERICAL ANGLE
      (( 0.0))
INPUT 2ND FIBER SPHERICAL ANGLE
      (( 0.0))

INPUT NUMBER OF JUNCTIONS AT LOCATION   1   2   2
      (( 1 ))
INPUT MATL. NO. 1 AT       1   2   2
      (( 2 ))
INPUT 1ST FIBER SPHERICAL ANGLE
      (( 0.0))
INPUT 2ND FIBER SPHERICAL ANGLE
      (( 0.0))
INPUT MATL. NO. 2 AT       1   2   2
      (( 1 ))
INPUT 1ST FIBER SPHERICAL ANGLE
      ((90.0))
INPUT 2ND FIBER SPHERICAL ANGLE
      (( 0.0))
INPUT 1ST MATL. VOLUME FRACTION
      (( 0.5))
INPUT 1ST INTERFACIAL NORMAL ANGLE
      (( 0.0))
INPUT 2ND INTERFACIAL NORMAL ANGLE
      (( 0.0))

INPUT NUMBER OF JUNCTIONS AT LOCATION   2   1   1
      (( 0 ))
SPECIFY THE CURRENT MATL. ID. NO.
      (( 2 ))
INPUT 1ST FIBER SPHERICAL ANGLE
      (( 0.0))
INPUT 2ND FIBER SPHERICAL ANGLE
      (( 0.0))

INPUT NUMBER OF JUNCTIONS AT LOCATION   2   1   2
      (( 0 ))
SPECIFY THE CURRENT MATL. ID. NO.
      (( 1 ))
INPUT 1ST FIBER SPHERICAL ANGLE
      ((90.0))
INPUT 2ND FIBER SPHERICAL ANGLE
      (( 0.0))

INPUT NUMBER OF JUNCTIONS AT LOCATION   2   2   1
      (( 0 ))
SPECIFY THE CURRENT MATL. ID. NO.
      (( 1 ))
INPUT 1ST FIBER SPHERICAL ANGLE
      (( 0.0))
INPUT 2ND FIBER SPHERICAL ANGLE
      (( 0.0))

```

INPUT NUMBER OF JUNCTIONS AT LOCATION 2 2 2
 ((0))
 SPECIFY THE CURRENT MATL. ID. NO.
 ((1))
 INPUT 1ST FIBER SPHERICAL ANGLE
 ((90.0))
 INPUT 2ND FIBER SPHERICAL ANGLE
 ((0.0))

ELASTIC CONSTANTS OF THE COMPOSITE

EX,EY,EZ = 5459114.00 7546501.50 3438649.50
 MUYZ,MUXZ,MUXY = 0.1276 0.1300 0.0544
 MUZY,MUZX,MUYX = 0.0581 0.0819 0.0752

INPUT APPLIED STRESSES IN X,Y,Z COORDINATES

INPUT X NORMAL STRESS
 ((1000.0))
 INPUT Y NORMAL STRESS
 ((0.0))
 INPUT Z NORMAL STRESS
 ((0.0))
 INPUT YZ SHEAR STRESS
 ((0.0))
 INPUT XZ SHEAR STRESS
 ((0.0))
 INPUT XY SHEAR STRESS
 ((0.0))

STRESSES IN ELEMENT NO. 1 1 1
 MATERIAL NO. 1
 NORMAL 1,2,3 -377.26 32.66 179.47
 SHEAR 23,13,12 -0.49 0.70 0.48
 MINIMUM MARGIN OF SAFETY IS 0.9976

STRESSES IN ELEMENT NO. 1 1 1
 MATERIAL NO. 2
 NORMAL 1,2,3 179.47 86.62 81.30
 SHEAR 23,13,12 0.12 -0.70 0.49
 MINIMUM MARGIN OF SAFETY IS 0.9759

STRESSES IN ELEMENT NO. 1 1 2
 MATERIAL NO. 1
 NORMAL 1,2,3 -348.27 66.55 292.88
 SHEAR 23,13,12 -5.62 -5.46 0.35
 MINIMUM MARGIN OF SAFETY IS 0.9976

STRESSES IN ELEMENT NO. 1 1 2
 MATERIAL NO. 1
 NORMAL 1,2,3 -96.79 292.88 50.10
 SHEAR 23,13,12 -5.46 0.35 -5.62
 MINIMUM MARGIN OF SAFETY IS 0.9990

STRESSES IN ELEMENT NO. 1 2 1
 MATERIAL NO. 1

NORMAL 1,2,3	3337.00	62.39	47.98
SHEAR 23,13,12	-0.30	1.68	1.93
MINIMUM MARGIN OF SAFETY IS		0.9816	

STRESSES IN ELEMENT NO. 1 2 2			
MATERIAL NO.	2		
NORMAL 1,2,3	190.64	91.97	86.67
SHEAR 23,13,12	-0.11	1.15	-1.28
MINIMUM MARGIN OF SAFETY IS		0.9744	

STRESSES IN ELEMENT NO. 1 2 2			
MATERIAL NO.	1		
NORMAL 1,2,3	-135.34	190.64	17.88
SHEAR 23,13,12	-1.15	-0.42	1.28
MINIMUM MARGIN OF SAFETY IS		0.9990	

STRESSES IN ELEMENT NO. 2 1 1			
MATERIAL NO.	2		
NORMAL 1,2,3	134.54	61.80	55.90
SHEAR 23,13,12	-0.30	-0.54	0.21
MINIMUM MARGIN OF SAFETY IS		0.9813	

STRESSES IN ELEMENT NO. 2 1 2			
MATERIAL NO.	1		
NORMAL 1,2,3	-106.32	282.81	49.86
SHEAR 23,13,12	-5.07	-0.75	-5.87
MINIMUM MARGIN OF SAFETY IS		0.9990	

STRESSES IN ELEMENT NO. 2 2 1			
MATERIAL NO.	1		
NORMAL 1,2,3	3307.09	61.25	44.10
SHEAR 23,13,12	0.81	0.53	1.53
MINIMUM MARGIN OF SAFETY IS		0.9818	

STRESSES IN ELEMENT NO. 2 2 2			
MATERIAL NO.	1		
NORMAL 1,2,3	-101.96	275.56	46.96
SHEAR 23,13,12	-1.68	1.24	2.88
MINIMUM MARGIN OF SAFETY IS		0.9990	

Appendix F

(FORTRAN Program Listing)

PROGRAM REPLACE

```

C
C
C      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C      CC  PROGRAM TO COMPUTE THE 3-D INTERNAL STRESSES  CC
C      CC      IN A UNIT CELL OF AN INCLUSION ARRAY      CC
C      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

```

SPECIFICATION STATEMENTS

```

C
C      PARAMETER (MM=10,MMM=300,NNN=400)
C      REAL MP(8,10),KS(24,24)
C      DIMENSION SK(24,24),UVWS(24)
C      DIMENSION KA(7),LC(24)
C      DIMENSION TS(6),TSS(6),TS1(6),TS2(6),ST1(6),ST2(6)
C      DIMENSION SIG(6),STN(6)
C      DIMENSION PR(6,6),RP(6,6),T(6,6),BIG(6,6)
C      DIMENSION DD(6,6),DD1(6,6),DD2(6,6)
C      DIMENSION SS(6,6),SS1(6,6),SS2(6,6)
C      DIMENSION SSS1(6,6),SSS2(6,6)
C      DIMENSION BM(6,24),DB(6,24)
C      DIMENSION PROP(MM,10),DX(MM),DY(MM),DZ(MM)
C      DIMENSION FDX(MM+1),FDY(MM+1),FDZ(MM+1)
C      DIMENSION FB(NNN,7),FS(MMM,7),FT(6,7),TF(6,7),FTT(6,6)
C      DIMENSION UVW(NNN,6),VU(NNN)
C      DIMENSION MN1(MM,MM,MM),MN2(MM,MM,MM),MN3(MM,MM,MM)
C      DIMENSION FV1(MM,MM,MM),FV2(MM,MM,MM),NJC(MM,MM,MM)
C      DIMENSION ANG1A(MM,MM,MM),ANG2A(MM,MM,MM),ANG3A(MM,MM,MM)
C      DIMENSION ANG1B(MM,MM,MM),ANG2B(MM,MM,MM),ANG3B(MM,MM,MM)
C      DIMENSION AGN1A(MM,MM,MM),AGN2A(MM,MM,MM)
C      DIMENSION AGN1B(MM,MM,MM),AGN2B(MM,MM,MM)
C      REAL KB(NNN,NNN),KM(MMM,NNN),KN(MMM,MMM)

```

```

C
C      BUILT IN MATERIAL PROPERTY DATA
C

```

```

      DATA (KA(I),I=1,7)/10,6,6,1,6,0,0/,
1  (MP(1,I),I=1,10)/18.E6,1.5E6,.23,.30,.7E6,.7E6,.01,.01,.01,.01/,
2  (MP(2,I),I=1,10)/2.1E7,1.7E6,.23,.30,.7E6,.7E6,.01,.01,.01,.01/,
3  (MP(3,I),I=1,10)/3.0E7,1.7E6,.23,.30,.7E6,.7E6,.01,.01,.01,.01/,
4  (MP(4,I),I=1,10)/1.0E7,1.5E6,.25,.35,.7E6,.7E6,.01,.01,.01,.01/,
5  (MP(5,I),I=1,10)/1.2E7,1.5E6,.25,.35,.7E6,.7E6,.01,.01,.01,.01/,
6  (MP(6,I),I=1,10)/.5E6,.5E6,.35,.35,.18E6,.18E6,.01,.01,.01,.01/,
7  (MP(7,I),I=1,10)/1.E7,1.E7,.30,.30,.4E7,.4E7,.01,.01,.01,.01/,
8  (MP(8,I),I=1,10)/1.E7,1.E7,.25,.25,.4E7,.4E7,.01,.01,.01,.01/

```

```

C
C      INITIALIZE VARIABLES
C

```

```

      ISYM=0
      DO 10 I=1,MMM
      DO 10 J=1,7
10  FS(I,J)=0.0
      DO 12 I=1,MMM
      DO 12 J=1,MMM
12  KN(I,J)=0.0
      DO 15 I=1,6
      DO 15 J=1,7
      TF(I,J)=0.0
15  FT(I,J)=0.0
      WRITE(6,9100)
      READ(5,9030) NM
      WRITE(6,9899) NM

```

```

C
C      MATERIAL PROPERTY DATA INPUT
C

```



```

DO 18 I=1,NM
WRITE(6,9180)
READ(5,9030) M
WRITE(6,9899) M
IF(M.GT.8) THEN
WRITE(6,9120)
READ(5,9010) PROP(I,1)
WRITE(6,9130)
READ(5,9010) PROP(I,2)
WRITE(6,9140)
READ(5,9010) PROP(I,3)
WRITE(6,9150)
READ(5,9010) PROP(I,4)
WRITE(6,9160)
READ(5,9010) PROP(I,5)
WRITE(6,9170)
READ(5,9010) PROP(I,6)
WRITE(6,9175)
READ(5,9015) PROP(I,7)
WRITE(6,9177)
READ(5,9015) PROP(I,8)
WRITE(6,9178)
READ(5,9015) PROP(I,9)
WRITE(6,9179)
READ(5,9015) PROP(I,10)
END IF
IF(M.LE.8) THEN
DO 17 J=1,10
17 PROP(I,J)=MP(M,J)
END IF
18 CONTINUE
WRITE(6,9560)
WRITE(6,9190)
DO 19 I=1,NM
WRITE(6,9020) (PROP(I,J),J=1,6)
19 WRITE(6,9025) PROP(I,7),PROP(I,8),PROP(I,9),PROP(I,10)
C
C READ MESH GEOMETRY
C
WRITE(6,9560)
WRITE(6,9440)
READ(5,9000) XL
WRITE(6,9898) XL
WRITE(6,9080)
READ(5,9030) NBX
WRITE(6,9899) NBX
FDX(1)=0.0
FDX(NBX+1)=100.0
IF(NBX.LE.1) GO TO 25
DO 22 I=1,NBX-1
WRITE(6,9460) I+1
READ(5,9000) FDX(I+1)
22 WRITE(6,9898) FDX(I+1)
25 CONTINUE
WRITE(6,9560)
WRITE(6,9450)
READ(5,9000) YL
WRITE(6,9898) YL
WRITE(6,9090)
READ(5,9030) NBY
WRITE(6,9899) NBY
FDY(1)=0.0
FDY(NBY+1)=100.0
IF(NBY.LE.1) GO TO 35
DO 30 I=1,NBY-1
WRITE(6,9460) I+1

```

```

      READ(5,9000) FDY(I+1)
30  WRITE(6,9898) FDY(I+1)
35  CONTINUE
      WRITE(6,9560)
      WRITE(6,9455)
      READ(5,9000) ZL
      WRITE(6,9898) ZL
      WRITE(6,9095)
      READ(5,9030) NBZ
      WRITE(6,9899) NBZ
      FDZ(1)=0.0
      FDZ(NBZ+1)=100.0
      IF(NBZ.LE.1) GO TO 135
      DO 130 I=1,NBZ-1
      WRITE(6,9460) I+1
      READ(5,9000) FDZ(I+1)
130  WRITE(6,9898) FDZ(I+1)
135  CONTINUE
      NP=(NBX+1)*(NBY+1)*(NBZ+1)*3
      DO 230 I=1,NP
      DO 230 J=1,6
230  UVW(I,J)=0.0
      DO 250 I=1,NNN
      DO 240 J=1,7
240  FB(I,J)=0.0
      DO 250 J=1,NNN
250  KB(I,J)=0.0
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C   BEGIN OUTER DO LOOP OVER THE NO. OF BRICK ELEMENTS      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 2400 I=1,NBX
      DO 2400 J=1,NBY
      DO 2400 K=1,NBZ
C
C GET ELEMENT DIMENSIONS
C
      A=FDX(I+1)-FDX(I)
      A=A*XL/100.
      AA=0.5*A
      DX(1)=0.0
      DX(2)=A
      B=FDY(J+1)-FDY(J)
      B=B*YL/100.
      BB=0.5*B
      DY(1)=0.0
      DY(2)=B
      C=FDZ(K+1)-FDZ(K)
      C=C*ZL/100.
      CC=0.5*C
      DZ(1)=0.0
      DZ(2)=C
      VOL=A*B*C
C
      DO 260 II=1,24
      DO 260 JJ=1,24
260  KS(II,JJ)=0.0
C
C INPUT TYPE OF MATERIAL JUNCTION IN THE ELEMENT (0,1,OR 2)
C
      WRITE(6,9560)
      WRITE(6,9060) I,J,K
      READ(5,9030) NJC(I,J,K)
      WRITE(6,9899) NJC(I,J,K)
      JNC=NJC(I,J,K)

```

```

C
C INPUT MATERIAL TYPE AND FIBER DIRECTION ANGLES (ONLY BRANCH)
C
  IF(JNC.LT.1) THEN
    WRITE(6,9320)
    READ(5,9030) MN1(I,J,K)
    MN=MN1(I,J,K)
    WRITE(6,9899) MN1(I,J,K)
    WRITE(6,9480)
    READ(5,9000) A1
    ANGLA(I,J,K)=A1
    WRITE(6,9898) A1
    WRITE(6,9490)
    READ(5,9000) A2
    ANGLB(I,J,K)=A2
    WRITE(6,9898) A2
    CALL TRANS2(A1,A2,T)
C
C GET STRESS-STRAIN MATRIX (SS) IN MATL. COORD. SYSTEM
C
  CALL GETSS(MM,MN,PROP,SS)
C
C GET STRESS-STRAIN MATRIX (SS) IN GLOBAL COORDINATES
C
  DO 290 II=1,6
    DO 290 JJ=1,6
      SUM=0.0
      DO 280 KK=1,6
280 SUM=SUM+SS(II,KK)*T(KK,JJ)
290 PR(II,JJ)=SUM
C
  DO 300 II=1,6
    DO 300 JJ=1,6
      SUM=0.0
      DO 295 KK=1,6
295 SUM=SUM+T(KK,II)*PR(KK,JJ)
300 BIG(II,JJ)=SUM
  END IF
C
C INPUT MATERIAL TYPE AND FIBER DIRECTION ANGLES (FIRST BRANCH)
C
  IF(JNC.GE.1) THEN
    WRITE(6,9700) I,J,K
    READ(5,9030) MN1(I,J,K)
    WRITE(6,9899) MN1(I,J,K)
    MN=MN1(I,J,K)
    WRITE(6,9480)
    READ(5,9000) ANGLA(I,J,K)
    WRITE(6,9898) ANGLA(I,J,K)
    A1=ANGLA(I,J,K)
    WRITE(6,9490)
    READ(5,9000) ANGLB(I,J,K)
    WRITE(6,9898) ANGLB(I,J,K)
    A2=ANGLB(I,J,K)
C
C GET STRESS-STRAIN MATRIX (SS) IN MATL. COORD. SYSTEM
C
  CALL GETSS(MM,MN,PROP,SS)
C
C GET STRESS-STRAIN MATRIX (SS) IN GLOBAL COORDINATES
C
  CALL TRANS2(A1,A2,T)
  DO 400 II=1,6
    DO 400 JJ=1,6
      SUM=0.0
      DO 350 KK=1,6

```

```

350 SUM=SUM+SS(II, KK) * T(KK, JJ)
400 PR(II, JJ)=SUM
    DO 450 II=1, 6
    DO 450 JJ=1, 6
    SUM=0.0
    DO 440 KK=1, 6
440 SUM=SUM+T(KK, II) * PR(KK, JJ)
450 SS1(II, JJ)=SUM
C
C INPUT MATERIAL TYPE AND FIBER DIRECTION ANGLES (SECOND BRANCH)
C
    WRITE(6, 9710) I, J, K
    READ(5, 9030) MN2(I, J, K)
    WRITE(6, 9899) MN2(I, J, K)
    MN=MN2(I, J, K)
    WRITE(6, 9480)
    READ(5, 9000) ANG2A(I, J, K)
    WRITE(6, 9898) ANG2A(I, J, K)
    A1=ANG2A(I, J, K)
    WRITE(6, 9490)
    READ(5, 9000) ANG2B(I, J, K)
    WRITE(6, 9898) ANG2B(I, J, K)
    A2=ANG2B(I, J, K)
C
C GET STRESS-STRAIN MATRIX (SS) IN MATL. COORD. SYSTEM
C
    CALL GETSS(MM, MN, PROP, SS)
C
C GET STRESS-STRAIN MATRIX (SS) IN GLOBAL COORDINATES
C
    CALL TRANS2(A1, A2, T)
    DO 500 II=1, 6
    DO 500 JJ=1, 6
    SUM=0.0
    DO 490 KK=1, 6
490 SUM=SUM+SS(II, KK) * T(KK, JJ)
500 PR(II, JJ)=SUM
    DO 550 II=1, 6
    DO 550 JJ=1, 6
    SUM=0.0
    DO 540 KK=1, 6
540 SUM=SUM+T(KK, II) * PR(KK, JJ)
550 SS2(II, JJ)=SUM
C
C INPUT FIRST BRANCH VOL. FRACTION AND INTERFACE NORMAL ANGLES
C
    WRITE(6, 9720)
    READ(5, 9000) FV1(I, J, K)
    WRITE(6, 9898) FV1(I, J, K)
    V1=FV1(I, J, K)
    V2=1.0-V1
    WRITE(6, 9485)
    READ(5, 9000) AGN1A(I, J, K)
    WRITE(6, 9898) AGN1A(I, J, K)
    A1=AGN1A(I, J, K)
    WRITE(6, 9495)
    READ(5, 9000) AGN1B(I, J, K)
    WRITE(6, 9898) AGN1B(I, J, K)
    A2=AGN1B(I, J, K)
C
C GET STRESS-STRAIN MATRICES (SS) IN INTERFACIAL COORDINATES
C
    CALL TRANS1(A1, A2, T)
    DO 600 II=1, 6
    DO 600 JJ=1, 6
    SUM=0.0

```

```

      TUM=0.0
      DO 590 KK=1,6
      SUM=SUM+SS1(II,KK)*T(KK,JJ)
590  TUM=TUM+SS2(II,KK)*T(KK,JJ)
      RP(II,JJ)=SUM
600  PR(II,JJ)=TUM
      DO 650 II=1,6
      DO 650 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 640 KK=1,6
      SUM=SUM+T(KK,II)*RP(KK,JJ)
640  TUM=TUM+T(KK,II)*PR(KK,JJ)
      SSS1(II,JJ)=SUM
650  SSS2(II,JJ)=TUM
C
C DO REPLACEMENT MATERIAL SUBSTITUTION AT FIRST JUNCTION
C
      CALL GETDD(SSS1,SSS2,V1,V2,DD1,DD2)
      DO 750 II=1,6
      DO 750 JJ=1,6
      DUM=0.0
      DO 740 KK=1,6
740  DUM=DUM+V1*SSS1(II,KK)*DD1(KK,JJ)+V2*SSS2(II,KK)*DD2(KK,JJ)
750  DD(II,JJ)=DUM
C
C GET REPLACEMENT STRESS-STRAIN MATRIX (SS) IN GLOBAL COORDINATES
C
      CALL TRANS2(A1,A2,T)
      DO 790 II=1,6
      DO 790 JJ=1,6
      SUM=0.0
      DO 780 KK=1,6
780  SUM=SUM+DD(II,KK)*T(KK,JJ)
790  PR(II,JJ)=SUM
      DO 850 II=1,6
      DO 850 JJ=1,6
      SUM=0.0
      DO 840 KK=1,6
840  SUM=SUM+T(KK,II)*PR(KK,JJ)
850  BIG(II,JJ)=SUM
      END IF
C
C INPUT MATERIAL TYPE AND FIBER DIRECTION ANGLES (THIRD BRANCH)
C
      IF(JNC.EQ.2) THEN
      WRITE(6,9715) I,J,K
      READ(5,9030) MN3(I,J,K)
      WRITE(6,9899) MN3(I,J,K)
      MN=MN3(I,J,K)
      WRITE(6,9480)
      READ(5,9000) ANG3A(I,J,K)
      WRITE(6,9898) ANG3A(I,J,K)
      A1=ANG3A(I,J,K)
      WRITE(6,9490)
      READ(5,9000) ANG3B(I,J,K)
      WRITE(6,9898) ANG3B(I,J,K)
      A2=ANG3B(I,J,K)
C
C GET STRESS-STRAIN MATRIX (SS) IN MATL. COORD. SYSTEM
C
      CALL GETSS(MM,MN,PROP,SS)
C
C GET STRESS-STRAIN MATRIX (SS) IN GLOBAL COORDINATES
C
      CALL TRANS2(A1,A2,T)

```

```

      DO 1400 II=1,6
      DO 1400 JJ=1,6
      SUM=0.0
      DO 1350 KK=1,6
1350 SUM=SUM+SS(II, KK)*T(KK, JJ)
1400 PR(II, JJ)=SUM
      DO 1450 II=1,6
      DO 1450 JJ=1,6
      SUM=0.0
      DO 1440 KK=1,6
1440 SUM=SUM+T(KK, II)*PR(KK, JJ)
1450 SS1(II, JJ)=SUM
      DO 1550 II=1,6
      DO 1550 JJ=1,6
1550 SS2(II, JJ)=BIG(II, JJ)
C
C INPUT LAST BRANCH VOL. FRACTION AND INTERFACE NORMAL ANGLES
C
      WRITE(6, 9720)
      READ(5, 9000) FV2(I, J, K)
      WRITE(6, 9898) FV2(I, J, K)
      V1=FV2(I, J, K)
      V2=1.0-V1
      WRITE(6, 9480)
      READ(5, 9000) AGN2A(I, J, K)
      WRITE(6, 9898) AGN2A(I, J, K)
      A1=AGN2A(I, J, K)
      WRITE(6, 9490)
      READ(5, 9000) AGN2B(I, J, K)
      WRITE(6, 9898) AGN2B(I, J, K)
      A2=AGN2B(I, J, K)
C
C GET STRESS-STRAIN MATRICES (SS) IN INTERFACIAL COORDINATES
C
      CALL TRANS1(A1, A2, T)
      DO 1600 II=1,6
      DO 1600 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 1590 KK=1,6
      SUM=SUM+SS1(II, KK)*T(KK, JJ)
1590 TUM=TUM+SS2(II, KK)*T(KK, JJ)
      RP(II, JJ)=SUM
1600 PR(II, JJ)=TUM
      DO 1650 II=1,6
      DO 1650 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 1640 KK=1,6
      SUM=SUM+T(KK, II)*RP(KK, JJ)
1640 TUM=TUM+T(KK, II)*PR(KK, JJ)
      SSS1(II, JJ)=SUM
1650 SSS2(II, JJ)=TUM
C
C DO REPLACEMENT MATERIAL SUBSTITUTION AT SECOND JUNCTION
C
      CALL GETDD(SSS1, SSS2, V1, V2, DD1, DD2)
      DO 1750 II=1,6
      DO 1750 JJ=1,6
      DUM=0.0
      DO 1740 KK=1,6
1740 DUM=DUM+V1*SSS1(II, KK)*DD1(KK, JJ)+V2*SSS2(II, KK)*DD2(KK, JJ)
1750 DD(II, JJ)=DUM
C
C GET REPLACEMENT STRESS-STRAIN MATRIX (SS) IN GLOBAL COORDINATES
C

```

```

      CALL TRANS2(A1,A2,T)
      DO 1790 II=1,6
      DO 1790 JJ=1,6
      SUM=0.0
      DO 1780 KK=1,6
1780 SUM=SUM+DD(II, KK)*T(KK, JJ)
1790 PR(II, JJ)=SUM
      DO 1850 II=1,6
      DO 1850 JJ=1,6
      SUM=0.0
      DO 1840 KK=1,6
1840 SUM=SUM+T(KK, II)*PR(KK, JJ)
1850 BIG(II, JJ)=SUM
      END IF
1900 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C BEGIN INTEGRATION SCHEME TO GET ELEMENT STIFFNESS MATRIX C
C INNER DO LOOP OVER EACH INTEGRATION POINT BEGINS HERE C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 2000 II=1,2
      DO 2000 JJ=1,2
      DO 2000 KK=1,2
C
      DO 1920 III=1,6
      DO 1920 JJJ=1,24
1920 BM(III, JJJ)=0.0
C
C SUBCELL GEOMETRY CALCULATIONS
C
      X=0.57735*AA
      IF(II.EQ.1) X=-X
      Y=0.57735*BB
      IF(JJ.EQ.1) Y=-Y
      Z=0.57735*CC
      IF(KK.EQ.1) Z=-Z
C
C DO GAUSSIAN INTEGRATION SCHEME
C
      CALL GETB(AA, BB, CC, X, Y, Z, BM)
      DO 1930 III=1,6
      DO 1930 JJJ=1,24
      DB(III, JJJ)=0.0
      DO 1930 KKK=1,6
1930 DB(III, JJJ)=DB(III, JJJ)+BIG(III, KKK)*BM(KKK, JJJ)
      DO 1950 III=1,24
      DO 1950 JJJ=1,24
      DO 1950 KKK=1,6
1950 KS(III, JJJ)=KS(III, JJJ)+BM(KKK, III)*DB(KKK, JJJ)
2000 CONTINUE
      DO 2040 III=1,24
      DO 2040 JJJ=1,24
2040 KS(III, JJJ)=KS(III, JJJ)*VOL/8.0
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C END OF INNER DO LOOP OVER INTEGRATION POINTS C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C PUT SMALL STIF. MATRIX (KS) INTO BIG STIF. MATRIX (KB)
C
      DO 2050 II=1,24
      DO 2050 JJ=1,24
2050 SK(II, JJ)=KS(II, JJ)
      LC(1)=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3+1
      LC(7)=LC(1)+(NBZ+1)*3

```

```

      LC(13)=LC(1)+(NBZ+1)*(NBY+1)*3
      LC(19)=LC(13)+(NBZ+1)*3
      DO 2200 KK=2,6
      LC(KK)=LC(KK-1)+1
      LC(KK+6)=LC(KK+5)+1
      LC(KK+12)=LC(KK+11)+1
2200  LC(KK+18)=LC(KK+17)+1
      DO 2300 II=1,24
      III=LC(II)
      DO 2300 JJ=1,24
      JJJ=LC(JJ)
2300  KB(III,JJJ)=KB(III,JJJ)+SK(II,JJ)
2400  CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  END OF OUTER DO LOOP ON NO. OF ELEMENTS IN UNIT CELL C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C  CALC. DISP. VECTORS FOR 6 HOMOGENEOUS UNIT STRAIN CASES
C
      DO 2420 I=1,NBX+1
      DO 2420 J=1,NBY+1
      DO 2420 K=1,NBZ+1
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
      UVW(L+1,1)=FDX(I)*XL/100.0
      UVW(L+2,2)=FDY(J)*YL/100.0
      UVW(L+3,3)=FDZ(K)*ZL/100.0
      UVW(L+2,4)=FDZ(K)*ZL/100.0
      UVW(L+1,5)=FDZ(K)*ZL/100.0
2420  UVW(L+1,6)=FDY(J)*YL/100.0
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  BEGIN NORMAL STRAIN ANALYSIS C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      GO TO 2460
2440  ISYM=1
      DO 2445 I=1,MMM
      DO 2445 J=1,MMM
2445  KN(I,J)=0.0
      DO 2447 I=1,MMM
      DO 2447 J=1,NNN
2447  KM(I,J)=0.0
      DO 2450 I=1,NP
      DO 2450 J=1,7
2450  FB(I,J)=0.0
2460  CONTINUE
C
C  USE ZERO FORCE CONDITIONS TO ELIMINATE INNER FORCES
C
      IN=0
      IF((NBX.GT.1).AND.(NBY.GT.1).AND.(NBZ.GT.1)) THEN
      DO 2510 I=2,NBX
      DO 2510 J=2,NBY
      DO 2510 K=2,NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
      DO 2500 M=1,NP
      KM(IN+1,M)=KB(L+1,M)
      KM(IN+2,M)=KB(L+2,M)
2500  KM(IN+3,M)=KB(L+3,M)
2510  IN=IN+3
      END IF
C
C  USE ZERO FORCE B.C.S ON Z-NORMAL FACES
C
      IF((NBX.GT.1).AND.(NBY.GT.1)) THEN
      DO 2530 I=2,NBX

```



```

DO 2530 J=2,NBY
L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1))*3
LL=L+NBZ*3
DO 2525 M=1,NP
KM(IN+1,M)=KB(L+1,M)
KM(IN+2,M)=KB(L+2,M)
KM(IN+3,M)=KB(LL+1,M)
2525 KM(IN+4,M)=KB(LL+2,M)
2530 IN=IN+4
END IF

C
C USE ZERO FORCE B.C.S ON Y-NORMAL FACES
C
IF((ISYM.EQ.0).AND.(NBX.GT.1).AND.(NBZ.GT.1)) THEN
DO 2555 I=2,NBX
DO 2555 K=2,NBZ
L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
LL=L+(NBZ+1)*(NBY)*3
DO 2550 M=1,NP
KM(IN+1,M)=KB(L+1,M)
KM(IN+2,M)=KB(L+3,M)
KM(IN+3,M)=KB(LL+1,M)
2550 KM(IN+4,M)=KB(LL+3,M)
2555 IN=IN+4
END IF
IF((ISYM.EQ.1).AND.(NBX.GT.1).AND.(NBZ.GT.0)) THEN
DO 2570 I=2,NBX
DO 2570 K=1,NBZ+1
L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
LL=L+(NBZ+1)*(NBY)*3
DO 2565 M=1,NP
KM(IN+1,M)=KB(L+2,M)
2565 KM(IN+2,M)=KB(LL+2,M)
2570 IN=IN+2
END IF

C
C USE ZERO FORCE B.C.S ON X-NORMAL FACES
C
IF((ISYM.EQ.0).AND.(NBY.GT.1).AND.(NBZ.GT.1)) THEN
DO 2585 J=2,NBY
DO 2585 K=2,NBZ
L=((NBZ+1)*(J-1)+(K-1))*3
LL=L+((NBZ+1)*(NBY+1)*NBX)*3
DO 2580 M=1,NP
KM(IN+1,M)=KB(L+2,M)
KM(IN+2,M)=KB(L+3,M)
KM(IN+3,M)=KB(LL+2,M)
2580 KM(IN+4,M)=KB(LL+3,M)
2585 IN=IN+4
END IF
IF((ISYM.EQ.1).AND.(NBY.GT.1).AND.(NBZ.GT.0)) THEN
DO 2600 J=2,NBY
DO 2600 K=1,NBZ+1
L=((NBZ+1)*(J-1)+(K-1))*3
LL=L+((NBZ+1)*(NBY+1)*NBX)*3
DO 2595 M=1,NP
KM(IN+1,M)=KB(L+1,M)
2595 KM(IN+2,M)=KB(LL+1,M)
2600 IN=IN+2
END IF

C
C USE ZERO FORCE B.C. ON Z-PARALLEL EDGES
C
IF((ISYM.EQ.0).AND.(NBZ.GT.1)) THEN
DO 2630 K=2,NBZ
L=(K-1)*3

```

```

      LL=L+(NBZ+1)*NBY*3
      LLL=L+(NBZ+1)*(NBY+1)*NBX*3
      LLLL=LL+(NBZ+1)*(NBY+1)*NBX*3
      DO 2620 M=1,NP
2620  KM(IN+1,M)=KB(L+3,M)+KB(LL+3,M)+KB(LLL+3,M)+KB(LLLL+3,M)
2630  IN=IN+1
      END IF

```

C
C USE ZERO FORCE B.C. ON Y-PARALLEL EDGES

```

C
      IF((ISYM.EQ.0).AND.(NBY.GT.1)) THEN
      DO 2680 J=2,NBY
      L=(NBZ+1)*(J-1)*3
      LL=L+NBZ*3
      LLL=L+(NBZ+1)*(NBY+1)*NBX*3
      LLLL=LL+(NBZ+1)*(NBY+1)*NBX*3
      DO 2670 M=1,NP
2670  KM(IN+1,M)=KB(L+2,M)+KB(LL+2,M)+KB(LLL+2,M)+KB(LLLL+2,M)
2680  IN=IN+1
      END IF

```

C
C USE ZERO FORCE B.C. ON X-PARALLEL EDGES

```

C
      IF((ISYM.EQ.0).AND.(NBX.GT.1)) THEN
      DO 2710 I=2,NBX
      L=(NBZ+1)*(NBY+1)*(I-1)*3
      LL=L+NBZ*3
      LLL=L+(NBZ+1)*NBY*3
      LLLL=LLL+NBZ*3
      DO 2700 M=1,NP
2700  KM(IN+1,M)=KB(L+1,M)+KB(LL+1,M)+KB(LLL+1,M)+KB(LLLL+1,M)
2710  IN=IN+1
      END IF

```

C
 IJ=IN

```

C
      IF(IJ.LT.1) GO TO 2745
9085  FORMAT(1H,6F12.0)
      DO 2730 I=1,IJ
      DO 2730 J=1,7
2730  FS(I,J)=0.0
      DO 2740 I=1,6
      DO 2735 J=1,IN
      DO 2735 K=1,NP
2735  FS(J,7)=FS(J,7)+KM(J,K)*UVW(K,I)
      DO 2740 J=1,IJ
      FS(J,I)=FS(J,7)
2740  FS(J,7)=0.0
2745  CONTINUE
      INN=IN
      IN=0

```

C
C RETAIN INTERNAL DISPLACEMENTS

```

C
      IF((NBX.GT.1).AND.(NBY.GT.1).AND.(NBZ.GT.1)) THEN
      DO 2755 I=2,NBX
      DO 2755 J=2,NBY
      DO 2755 K=2,NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
      DO 2750 M=1,INN
      KN(M,IN+1)=KM(M,L+1)
      KN(M,IN+2)=KM(M,L+2)
2750  KN(M,IN+3)=KM(M,L+3)
2755  IN=IN+3
      END IF

```

C

C USE DISPLACEMENT B.C.S ON Z-NORMAL FACES

```
C
  IF ((NBX.GT.1).AND.(NBY.GT.1)) THEN
    DO 2770 I=2,NBX
    DO 2770 J=2,NBY
    L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1))*3
    LL=L+NBZ*3
    DO 2765 M=1,INN
    KN(M,IN+1)=KM(M,L+1)
    KN(M,IN+2)=KM(M,L+2)
    KN(M,IN+3)=KM(M,LL+1)
2765 KN(M,IN+4)=KM(M,LL+2)
2770 IN=IN+4
  END IF
```

C USE DISPLACEMENT B.C.S ON Y-NORMAL FACES

```
C
  IF ((ISYM.EQ.0).AND.(NBX.GT.1).AND.(NBZ.GT.1)) THEN
    DO 2779 I=2,NBX
    DO 2779 K=2,NBZ
    L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
    LL=L+(NBZ+1)*(NBY)*3
    DO 2777 M=1,INN
    KN(M,IN+1)=KM(M,L+1)
    KN(M,IN+2)=KM(M,L+3)
    KN(M,IN+3)=KM(M,LL+1)
2777 KN(M,IN+4)=KM(M,LL+3)
2779 IN=IN+4
  END IF
  IF ((ISYM.EQ.1).AND.(NBX.GT.1).AND.(NBZ.GT.0)) THEN
    DO 2786 I=2,NBX
    DO 2786 K=1,NBZ+1
    L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
    LL=L+(NBZ+1)*(NBY)*3
    DO 2784 M=1,INN
    KN(M,IN+1)=KM(M,L+2)
2784 KN(M,IN+2)=KM(M,LL+2)
2786 IN=IN+2
  END IF
```

C USE DISPLACEMENT B.C.S ON X-NORMAL FACES

```
C
  IF ((ISYM.EQ.0).AND.(NBY.GT.1).AND.(NBZ.GT.1)) THEN
    DO 2792 J=2,NBY
    DO 2792 K=2,NBZ
    L=((NBZ+1)*(J-1)+(K-1))*3
    LL=L+((NBZ+1)*(NBY+1)*NBX)*3
    DO 2790 M=1,INN
    KN(M,IN+1)=KM(M,L+2)
    KN(M,IN+2)=KM(M,L+3)
    KN(M,IN+3)=KM(M,LL+2)
2790 KN(M,IN+4)=KM(M,LL+3)
2792 IN=IN+4
  END IF
  IF ((ISYM.EQ.1).AND.(NBY.GT.1).AND.(NBZ.GT.0)) THEN
    DO 2796 J=2,NBY
    DO 2796 K=1,NBZ+1
    L=((NBZ+1)*(J-1)+(K-1))*3
    LL=L+((NBZ+1)*(NBY+1)*NBX)*3
    DO 2795 M=1,INN
    KN(M,IN+1)=KM(M,L+1)
2795 KN(M,IN+2)=KM(M,LL+1)
2796 IN=IN+2
  END IF
```

C USE DISPLACEMENT B.C. ON Z-PARALLEL EDGES

```

C
  IF ((ISYM.EQ.0).AND.(NBZ.GT.1)) THEN
    DO 2805 K=2,NBZ
      L=(K-1)*3
      LL=L+(NBZ+1)*NBY*3
      LLL=L+(NBZ+1)*(NBY+1)*NBX*3
      LLLL=LL+(NBZ+1)*(NBY+1)*NBX*3
      DO 2800 M=1,INN
2800  KN(M,IN+1)=KM(M,L+3)+KM(M,LL+3)+KM(M,LLL+3)+KM(M,LLLL+3)
2805  IN=IN+1
      END IF
C
C USE DISPLACEMENT B.C. ON Y-PARALLEL EDGES
C
  IF ((ISYM.EQ.0).AND.(NBY.GT.1)) THEN
    DO 2840 J=2,NBY
      L=(NBZ+1)*(J-1)*3
      LL=L+NBZ*3
      LLL=L+(NBZ+1)*(NBY+1)*NBX*3
      LLLL=LL+(NBZ+1)*(NBY+1)*NBX*3
      DO 2830 M=1,INN
2830  KN(M,IN+1)=KM(M,L+2)+KM(M,LL+2)+KM(M,LLL+2)+KM(M,LLLL+2)
2840  IN=IN+1
      END IF
C
C USE DISPLACEMENT B.C. ON X-PARALLEL EDGES
C
  IF ((ISYM.EQ.0).AND.(NBX.GT.1)) THEN
    DO 2880 I=2,NBX
      L=(NBZ+1)*(NBY+1)*(I-1)*3
      LL=L+NBZ*3
      LLL=L+(NBZ+1)*NBY*3
      LLLL=LLL+NBZ*3
      DO 2870 M=1,INN
2870  KN(M,IN+1)=KM(M,L+1)+KM(M,LL+1)+KM(M,LLL+1)+KM(M,LLLL+1)
2880  IN=IN+1
      END IF
C
C GET UNCONSTRAINED DISPLACEMENTS FOR UNIT STRAIN CASES
C
  CALL MATINV(KN,MMM,IJ,FS,7,7,DET)
C
C GET A COMPLETE SET OF TOTAL DISPLACEMENTS
C
  IN=0
C
C ON INTERIOR
C
  IF ((NBX.GT.1).AND.(NBY.GT.1).AND.(NBZ.GT.1)) THEN
    DO 2905 I=2,NBX
      DO 2905 J=2,NBY
        DO 2905 K=2,NBZ
          L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
          IF (ISYM.EQ.1) GO TO 2902
          DO 2900 M=1,3
            UVW(L+1,M)=UVW(L+1,M)-FS(IN+1,M)
            UVW(L+2,M)=UVW(L+2,M)-FS(IN+2,M)
2900    UVW(L+3,M)=UVW(L+3,M)-FS(IN+3,M)
            GO TO 2905
          2902 CONTINUE
          M=6
          UVW(L+1,M)=UVW(L+1,M)-FS(IN+1,M)
          UVW(L+2,M)=UVW(L+2,M)-FS(IN+2,M)
          UVW(L+3,M)=UVW(L+3,M)-FS(IN+3,M)
2905  IN=IN+3
        END IF
      END IF
    END IF
  END IF

```

```

C
C ON Z-NORMAL FACES
C
  IF ((NBX.GT.1).AND.(NBY.GT.1)) THEN
    DO 2930 I=2,NBX
    DO 2930 J=2,NBY
    L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1))*3
    LL=L+NBZ*3
    IF (ISYM.EQ.1) GO TO 2925
    DO 2920 M=1,3
    UVW(L+1,M)=UVW(L+1,M)-FS(IN+1,M)
    UVW(L+2,M)=UVW(L+2,M)-FS(IN+2,M)
    UVW(LL+1,M)=UVW(LL+1,M)-FS(IN+3,M)
2920 UVW(LL+2,M)=UVW(LL+2,M)-FS(IN+4,M)
    GO TO 2930
2925 CONTINUE
    M=6
    UVW(L+1,M)=UVW(L+1,M)-FS(IN+1,M)
    UVW(L+2,M)=UVW(L+2,M)-FS(IN+2,M)
    UVW(LL+1,M)=UVW(LL+1,M)-FS(IN+3,M)
    UVW(LL+2,M)=UVW(LL+2,M)-FS(IN+4,M)
2930 IN=IN+4
    END IF

C
C ON Y-NORMAL FACES
C
  IF ((ISYM.EQ.0).AND.(NBX.GT.1).AND.(NBZ.GT.1)) THEN
    DO 2950 I=2,NBX
    DO 2950 K=2,NBZ
    L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
    LL=L+(NBZ+1)*(NBY)*3
    DO 2945 M=1,3
    UVW(L+1,M)=UVW(L+1,M)-FS(IN+1,M)
    UVW(L+3,M)=UVW(L+3,M)-FS(IN+2,M)
    UVW(LL+1,M)=UVW(LL+1,M)-FS(IN+3,M)
2945 UVW(LL+3,M)=UVW(LL+3,M)-FS(IN+4,M)
2950 IN=IN+4
    END IF
    IF ((ISYM.EQ.1).AND.(NBX.GT.1).AND.(NBZ.GT.0)) THEN
      DO 2960 I=2,NBX
      DO 2960 K=1,NBZ+1
      L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
      LL=L+(NBZ+1)*(NBY)*3
      M=6
      UVW(L+2,M)=UVW(L+2,M)-FS(IN+1,M)
      UVW(LL+2,M)=UVW(LL+2,M)-FS(IN+2,M)
2960 IN=IN+2
    END IF

C
C ON X-NORMAL FACES
C
  IF ((ISYM.EQ.0).AND.(NBY.GT.1).AND.(NBZ.GT.1)) THEN
    DO 2980 J=2,NBY
    DO 2980 K=2,NBZ
    L=((NBZ+1)*(J-1)+(K-1))*3
    LL=L+((NBZ+1)*(NBY+1)*NBX)*3
    DO 2975 M=1,3
    UVW(L+2,M)=UVW(L+2,M)-FS(IN+1,M)
    UVW(L+3,M)=UVW(L+3,M)-FS(IN+2,M)
    UVW(LL+2,M)=UVW(LL+2,M)-FS(IN+3,M)
2975 UVW(LL+3,M)=UVW(LL+3,M)-FS(IN+4,M)
2980 IN=IN+4
    END IF
    IF ((ISYM.EQ.1).AND.(NBY.GT.1).AND.(NBZ.GT.0)) THEN
      DO 2996 J=2,NBY
      DO 2996 K=1,NBZ+1

```

```

      L=(NBZ+1)*(J-1)+(K-1))*3
      LL=L+(NBZ+1)*(NBY+1)*NBX*3
      M=6
      UVW(L+1,M)=UVW(L+1,M)-FS(IN+1,M)
      UVW(LL+1,M)=UVW(LL+1,M)-FS(IN+2,M)
2996  IN=IN+2
      END IF
C
C ON Z-PARALLEL EDGES
C
      IF((ISYM.EQ.0).AND.(NBZ.GT.1)) THEN
      DO 3010 K=2,NBZ
      L=(K-1)*3
      LL=L+(NBZ+1)*NBY*3
      LLL=L+(NBZ+1)*(NBY+1)*NBX*3
      LLLL=LL+(NBZ+1)*(NBY+1)*NBX*3
      DO 3000 M=1,3
      UVW(L+3,M)=UVW(L+3,M)-FS(IN+1,M)
      UVW(LL+3,M)=UVW(LL+3,M)-FS(IN+1,M)
      UVW(LL+3,M)=UVW(LL+3,M)-FS(IN+1,M)
      UVW(LL+3,M)=UVW(LL+3,M)-FS(IN+1,M)
3000  UVW(LL+3,M)=UVW(LL+3,M)-FS(IN+1,M)
3010  IN=IN+1
      END IF

```

```

C
C ON Y-PARALLEL EDGES
C
      IF((ISYM.EQ.0).AND.(NBY.GT.1)) THEN
      DO 3040 J=2,NBY
      L=(NBZ+1)*(J-1)*3
      LL=L+NBZ*3
      LLL=L+(NBZ+1)*(NBY+1)*NBX*3
      LLLL=LL+(NBZ+1)*(NBY+1)*NBX*3
      DO 3030 M=1,3
      UVW(L+2,M)=UVW(L+2,M)-FS(IN+1,M)
      UVW(LL+2,M)=UVW(LL+2,M)-FS(IN+1,M)
      UVW(LL+2,M)=UVW(LL+2,M)-FS(IN+1,M)
      UVW(LL+2,M)=UVW(LL+2,M)-FS(IN+1,M)
3030  UVW(LL+2,M)=UVW(LL+2,M)-FS(IN+1,M)
3040  IN=IN+1
      END IF

```

```

C
C ON X-PARALLEL EDGES
C
      IF((ISYM.EQ.0).AND.(NBX.GT.1)) THEN
      DO 3070 I=2,NBX
      L=(NBZ+1)*(NBY+1)*(I-1)*3
      LL=L+NBZ*3
      LLL=L+(NBZ+1)*NBY*3
      LLLL=LLL+NBZ*3
      DO 3060 M=1,3
      UVW(L+1,M)=UVW(L+1,M)-FS(IN+1,M)
      UVW(LL+1,M)=UVW(LL+1,M)-FS(IN+1,M)
      UVW(LL+1,M)=UVW(LL+1,M)-FS(IN+1,M)
      UVW(LL+1,M)=UVW(LL+1,M)-FS(IN+1,M)
3060  UVW(LL+1,M)=UVW(LL+1,M)-FS(IN+1,M)
3070  IN=IN+1
      END IF
      IF(ISYM.EQ.0) GO TO 2440

```

```

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C BEGIN ((GAMMA-YZ) = 1.0) SHEAR STRAIN ANALYSIS C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

```

```

      DO 3145 I=1,MMM
      DO 3145 J=1,MMM
3145  KN(I,J)=0.0
      DO 3147 I=1,MMM
      DO 3147 J=1,NNN

```

```

3147 KM(I,J)=0.0
      DO 3150 I=1,NP
      DO 3150 J=1,7
3150 FB(I,J)=0.0
C
C USE ZERO FORCE CONDITIONS TO ELIMINATE INNER FORCES
C
      IN=0
      IF((NBX.LE.1).OR.(NBY.LE.1).OR.(NBZ.LE.1)) GO TO 3165
      DO 3160 I=2,NBX
      DO 3160 J=2,NBY
      DO 3160 K=2,NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
      DO 3155 M=1,NP
      KM(IN+1,M)=KB(L+1,M)
      KM(IN+2,M)=KB(L+2,M)
3155 KM(IN+3,M)=KB(L+3,M)
3160 IN=IN+3
3165 CONTINUE
C
C USE ZERO FORCE B.C.S ON Z-NORMAL FACES
C
      IF((NBX.LE.0).OR.(NBY.LE.1)) GO TO 3240
      DO 3230 I=1,NBX+1
      DO 3230 J=2,NBY
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1))*3
      LL=L+NBZ*3
      DO 3220 M=1,NP
      KM(IN+1,M)=KB(L+3,M)
3220 KM(IN+2,M)=KB(LL+3,M)
3230 IN=IN+2
3240 CONTINUE
C
C USE ZERO FORCE B.C.S ON Y-NORMAL FACES
C
      IF((NBX.LE.0).OR.(NBZ.LE.1)) GO TO 3260
      DO 3255 I=1,NBX+1
      DO 3255 K=2,NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
      LL=L+(NBZ+1)*(NBY)*3
      DO 3250 M=1,NP
      KM(IN+1,M)=KB(L+2,M)
3250 KM(IN+2,M)=KB(LL+2,M)
3255 IN=IN+2
3260 CONTINUE
C
C USE ZERO FORCE B.C.S ON X-NORMAL FACES
C
      IF((NBY.LE.1).OR.(NBZ.LE.1)) GO TO 3290
      DO 3280 J=2,NBY
      DO 3280 K=2,NBZ
      L=((NBZ+1)*(J-1)+(K-1))*3
      LL=L+((NBZ+1)*(NBY+1)*NBX)*3
      DO 3270 M=1,NP
      KM(IN+1,M)=KB(L+2,M)
      KM(IN+2,M)=KB(L+3,M)
      KM(IN+3,M)=KB(LL+2,M)
3270 KM(IN+4,M)=KB(LL+3,M)
3280 IN=IN+4
3290 CONTINUE
3320 CONTINUE
C
      IJ=IN
C
C GET UNCONSTRAINED NODAL FORCES
C

```

```

      IF(IJ.LT.1) GO TO 3345
      DO 3330 I=1, IJ
      J=7
3330  FS(I,J)=0.0
      I=4
      DO 3335 J=1, IN
      DO 3335 K=1, NP
3335  FS(J,7)=FS(J,7)+KM(J,K)*UVW(K,I)
      DO 3340 J=1, IJ
      FS(J,I)=FS(J,7)
3340  FS(J,7)=0.0
3345  CONTINUE
C
C  RETAIN INTERNAL DISPLACEMENTS
C
      INN=IN
      IN=0
      IF(IJ.LT.1) GO TO 3595
      IF((NBX.LE.1).OR.(NBY.LE.1).OR.(NBZ.LE.1)) GO TO 3400
      DO 3390 I=2, NBX
      DO 3390 J=2, NBY
      DO 3390 K=2, NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
      DO 3380 M=1, INN
      KN(M,IN+1)=KM(M,L+1)
      KN(M,IN+2)=KM(M,L+2)
3380  KN(M,IN+3)=KM(M,L+3)
3390  IN=IN+3
3400  CONTINUE
C
C  USE DISPLACEMENT B.C.S ON Z-NORMAL FACES
C
      IF((NBX.LE.0).OR.(NBY.LE.1)) GO TO 3440
      DO 3430 I=1, NBX+1
      DO 3430 J=2, NBY
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1))*3
      LL=L+NBZ*3
      DO 3420 M=1, INN
      KN(M,IN+1)=KM(M,L+3)
3420  KN(M,IN+2)=KM(M,LL+3)
3430  IN=IN+2
3440  CONTINUE
C
C  USE DISPLACEMENT B.C.S ON Y-NORMAL FACES
C
      IF((NBX.LE.0).OR.(NBZ.LE.1)) GO TO 3480
      DO 3475 I=1, NBX+1
      DO 3475 K=2, NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
      LL=L+(NBZ+1)*(NBY)*3
      DO 3470 M=1, INN
      KN(M,IN+1)=KM(M,L+2)
3470  KN(M,IN+2)=KM(M,LL+2)
3475  IN=IN+2
3480  CONTINUE
C
C  USE DISPLACEMENT B.C.S ON X-NORMAL FACES
C
      IF((NBY.LE.1).OR.(NBZ.LE.1)) GO TO 3494
      DO 3492 J=2, NBY
      DO 3492 K=2, NBZ
      L=((NBZ+1)*(J-1)+(K-1))*3
      LL=L+((NBZ+1)*(NBY+1)*NBX)*3
      DO 3490 M=1, INN
      KN(M,IN+1)=KM(M,L+2)
      KN(M,IN+2)=KM(M,L+3)

```



```

      KN(M, IN+3) = KM(M, LL+2)
3490 KN(M, IN+4) = KM(M, LL+3)
3492 IN = IN+4
3494 CONTINUE
C
3595 CONTINUE
C
C GET UNCONSTRAINED DISPLACEMENTS FOR UNIT STRAIN CASES
C
      IJ = IN
      CALL MATINV(KN, MMM, IJ, FS, 7, 7, DET)
C
C GET A COMPLETE SET OF TOTAL DISPLACEMENTS
C
C ON INTERIOR
C
      IN = 0
      IF((NBX.LE.1).OR.(NBY.LE.1).OR.(NBZ.LE.1)) GO TO 3610
      DO 3605 I=2, NBX
      DO 3605 J=2, NBY
      DO 3605 K=2, NBZ
      L = ((NBZ+1) * (NBY+1) * (I-1) + (NBZ+1) * (J-1) + (K-1)) * 3
      M = 4
      UVW(L+1, M) = UVW(L+1, M) - FS(IN+1, M)
      UVW(L+2, M) = UVW(L+2, M) - FS(IN+2, M)
      UVW(L+3, M) = UVW(L+3, M) - FS(IN+3, M)
3605 IN = IN+3
3610 CONTINUE
C
C ON Z-NORMAL FACES
C
      IF((NBX.LE.0).OR.(NBY.LE.1)) GO TO 3640
      DO 3630 I=1, NBX+1
      DO 3630 J=2, NBY
      L = ((NBZ+1) * (NBY+1) * (I-1) + (NBZ+1) * (J-1)) * 3
      LL = L + NBZ * 3
      M = 4
      UVW(L+3, M) = UVW(L+3, M) - FS(IN+1, M)
      UVW(LL+3, M) = UVW(LL+3, M) - FS(IN+2, M)
3630 IN = IN+2
3640 CONTINUE
C
C ON Y-NORMAL FACES
C
      IF((NBX.LE.0).OR.(NBZ.LE.1)) GO TO 3690
      DO 3680 I=1, NBX+1
      DO 3680 K=2, NBZ
      L = ((NBZ+1) * (NBY+1) * (I-1) + (K-1)) * 3
      LL = L + (NBZ+1) * (NBY) * 3
      M = 4
      UVW(L+2, M) = UVW(L+2, M) - FS(IN+1, M)
      UVW(LL+2, M) = UVW(LL+2, M) - FS(IN+2, M)
3680 IN = IN+2
3690 CONTINUE
C
C ON X-NORMAL FACES
C
      IF((NBY.LE.1).OR.(NBZ.LE.1)) GO TO 3790
      DO 3780 J=2, NBY
      DO 3780 K=2, NBZ
      L = ((NBZ+1) * (J-1) + (K-1)) * 3
      LL = L + ((NBZ+1) * (NBY+1) * NBX) * 3
      M = 4
      UVW(L+2, M) = UVW(L+2, M) - FS(IN+1, M)
      UVW(L+3, M) = UVW(L+3, M) - FS(IN+2, M)
      UVW(LL+2, M) = UVW(LL+2, M) - FS(IN+3, M)

```

```

3775 UVW(LL+3,M)-UVW(LL+3,M)-FS(IN+4,M)
3780 IN=IN+4
3790 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C BEGIN ((GAMMA-XZ) = 1.0) SHEAR STRAIN ANALYSIS C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 3845 I=1,MMM
      DO 3845 J=1,MMM
3845  KN(I,J)=0.0
      DO 3847 I=1,MMM
      DO 3847 J=1,NNN
3847  KM(I,J)=0.0
      DO 3850 I=1,NP
      DO 3850 J=1,7
3850  FB(I,J)=0.0
C
C USE ZERO FORCE CONDITIONS TO ELIMINATE INNER FORCES
C
      IN=0
      IF((NBX.LE.1).OR.(NBY.LE.1).OR.(NBZ.LE.1)) GO TO 3865
      DO 3860 I=2,NBX
      DO 3860 J=2,NBY
      DO 3860 K=2,NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
      DO 3855 M=1,NP
      KM(IN+1,M)=KB(L+1,M)
      KM(IN+2,M)=KB(L+2,M)
3855  KM(IN+3,M)=KB(L+3,M)
3860  IN=IN+3
3865  CONTINUE
C
C USE ZERO FORCE B.C.S ON Z-NORMAL FACES
C
      IF((NBX.LE.1).OR.(NBY.LE.0)) GO TO 3940
      DO 3930 I=2,NBX
      DO 3930 J=1,NBY+1
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1))*3
      LL=L+NBZ*3
      DO 3920 M=1,NP
      KM(IN+1,M)=KB(L+3,M)
3920  KM(IN+2,M)=KB(LL+3,M)
3930  IN=IN+2
3940  CONTINUE
C
C USE ZERO FORCE B.C.S ON Y-NORMAL FACES
C
      IF((NBX.LE.1).OR.(NBZ.LE.1)) GO TO 4050
      DO 3980 I=2,NBX
      DO 3980 K=2,NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
      LL=L+(NBZ+1)*(NBY)*3
      DO 3970 M=1,NP
      KM(IN+1,M)=KB(L+1,M)
      KM(IN+2,M)=KB(L+3,M)
      KM(IN+3,M)=KB(LL+1,M)
3970  KM(IN+4,M)=KB(LL+3,M)
3980  IN=IN+4
4050  CONTINUE
C
C USE ZERO FORCE B.C.S ON X-NORMAL FACES
C
      IF((NBY.LE.0).OR.(NBZ.LE.1)) GO TO 4150
      DO 4080 J=1,NBY+1
      DO 4080 K=2,NBZ

```

```

      L=((NBZ+1)*(J-1)+(K-1))*3
      LL=L+((NBZ+1)*(NBY+1)*NBX)*3
      DO 4070 M=1,NP
      KM(IN+1,M)=KB(L+1,M)
4070 KM(IN+2,M)=KB(LL+1,M)
4080 IN=IN+2
4150 CONTINUE
4320 CONTINUE
C
      IJ=IN
C
C GET UNCONSTRAINED NODAL FORCES
C
      IF(IJ.LT.1) GO TO 4345
      DO 4330 I=1,IJ
      J=7
4330 FS(I,J)=0.0
      I=5
      DO 4335 J=1,IN
      DO 4335 K=1,NP
4335 FS(J,7)=FS(J,7)+KM(J,K)*UVW(K,I)
      DO 4340 J=1,IJ
      FS(J,I)=FS(J,7)
4340 FS(J,7)=0.0
4345 CONTINUE
C
C RETAIN INTERNAL DISPLACEMENTS
C
      INN=IN
      IN=0
      IF(IJ.LT.1) GO TO 4595
      IF((NBX.LE.1).OR.(NBY.LE.1).OR.(NBZ.LE.1)) GO TO 4400
      DO 4390 I=2,NBX
      DO 4390 J=2,NBY
      DO 4390 K=2,NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
      DO 4380 M=1,INN
      KN(M,IN+1)=KM(M,L+1)
      KN(M,IN+2)=KM(M,L+2)
4380 KN(M,IN+3)=KM(M,L+3)
4390 IN=IN+3
4400 CONTINUE
C
C USE DISPLACEMENT B.C.S ON Z-NORMAL FACES
C
      IF((NBX.LE.1).OR.(NBY.LE.0)) GO TO 4440
      DO 4430 I=2,NBX
      DO 4430 J=1,NBY+1
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1))*3
      LL=L+NBZ*3
      DO 4420 M=1,INN
      KN(M,IN+1)=KM(M,L+3)
4420 KN(M,IN+2)=KM(M,LL+3)
4430 IN=IN+2
4440 CONTINUE
C
C USE DISPLACEMENT B.C.S ON Y-NORMAL FACES
C
      IF((NBX.LE.1).OR.(NBZ.LE.1)) GO TO 4480
      DO 4475 I=2,NBX
      DO 4475 K=2,NBZ
      L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
      LL=L+(NBZ+1)*(NBY)*3
      DO 4470 M=1,INN
      KN(M,IN+1)=KM(M,L+1)
      KN(M,IN+2)=KM(M,L+3)

```

```

      KN(M, IN+3) = KM(M, LL+1)
4470 KN(M, IN+4) = KM(M, LL+3)
4475 IN = IN+4
4480 CONTINUE
C
C USE DISPLACEMENT B.C.S ON X-NORMAL FACES
C
      IF((NBX.LE.0).OR.(NBZ.LE.1)) GO TO 4494
      DO 4492 J=1, NBY+1
      DO 4492 K=2, NBZ
      L = ((NBZ+1)*(J-1) + (K-1))*3
      LL = L + ((NBZ+1)*(NBY+1)*NBX)*3
      DO 4490 M=1, INN
      KN(M, IN+1) = KM(M, L+1)
4490 KN(M, IN+2) = KM(M, LL+1)
4492 IN = IN+2
4494 CONTINUE
C
4595 CONTINUE
C
C GET UNCONSTRAINED DISPLACEMENTS FOR UNIT STRAIN CASES
C
      IJ = IN
      CALL MATINV(KN, MMM, IJ, FS, 7, 7, DET)
C
C GET A COMPLETE SET OF TOTAL DISPLACEMENTS
C
C ON INTERIOR
C
      IN = 0
      IF((NBX.LE.1).OR.(NBY.LE.1).OR.(NBZ.LE.1)) GO TO 4610
      DO 4605 I=2, NBX
      DO 4605 J=2, NBY
      DO 4605 K=2, NBZ
      L = ((NBZ+1)*(NBY+1)*(I-1) + (NBZ+1)*(J-1) + (K-1))*3
      M = 5
      UVW(L+1, M) = UVW(L+1, M) - FS(IN+1, M)
      UVW(L+2, M) = UVW(L+2, M) - FS(IN+2, M)
      UVW(L+3, M) = UVW(L+3, M) - FS(IN+3, M)
4605 IN = IN+3
4610 CONTINUE
C
C ON Z-NORMAL FACES
C
      IF((NBX.LE.1).OR.(NBY.LE.0)) GO TO 4640
      DO 4630 I=2, NBX
      DO 4630 J=1, NBY+1
      L = ((NBZ+1)*(NBY+1)*(I-1) + (NBZ+1)*(J-1))*3
      LL = L + NBZ*3
      M = 5
      UVW(L+3, M) = UVW(L+3, M) - FS(IN+1, M)
      UVW(LL+3, M) = UVW(LL+3, M) - FS(IN+2, M)
4630 IN = IN+2
4640 CONTINUE
C
C ON Y-NORMAL FACES
C
      IF((NBX.LE.1).OR.(NBZ.LE.1)) GO TO 4690
      DO 4680 I=2, NBX
      DO 4680 K=2, NBZ
      L = ((NBZ+1)*(NBY+1)*(I-1) + (K-1))*3
      LL = L + (NBZ+1)*(NBY)*3
      M = 5
      UVW(L+1, M) = UVW(L+1, M) - FS(IN+1, M)
      UVW(L+3, M) = UVW(L+3, M) - FS(IN+2, M)
      UVW(LL+1, M) = UVW(LL+1, M) - FS(IN+3, M)

```

```

      UVW(LL+3,M) = UVW(LL+3,M) - FS(IN+4,M)
4680 IN = IN+4
4690 CONTINUE
C
C ON X-NORMAL FACES
C
      IF((NBY.LE.0).OR.(NBZ.LE.1)) GO TO 4790
      DO 4780 J=1,NBY+1
      DO 4780 K=2,NBZ
      L=((NBZ+1)*(J-1)+(K-1))*3
      LL=L+((NBZ+1)*(NBY+1)*NBX)*3
      M=5
      UVW(L+1,M) = UVW(L+1,M) - FS(IN+1,M)
      UVW(LL+1,M) = UVW(LL+1,M) - FS(IN+2,M)
4780 IN = IN+2
4790 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C COMPUTE NODAL FORCES AND ELASTIC CONSTANTS C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      IF(NP.LT.1) GO TO 4950
      DO 4900 I=1,NP
      DO 4900 J=1,6
      FB(I,J) = 0.0
      DO 4900 K=1,NP
4900 FB(I,J) = FB(I,J) + KB(I,K)*UVW(K,J)
4950 CONTINUE
C
C COMPUTE SIDE LOADS FOR EACH UNIT STRAIN CASE
C
      XNA = YL*ZL
      YNA = XL*ZL
      ZNA = XL*YL
      DO 5400 M=1,6
      DO 5250 J=1,NBY+1
      DO 5250 K=1,NBZ+1
      L=((NBZ+1)*(J-1)+(K-1))*3
      FT(1,M) = FT(1,M) - FB(L+1,M)
      FT(6,M) = FT(6,M) - FB(L+2,M)
5250 FT(5,M) = FT(5,M) - FB(L+3,M)
      FT(1,M) = FT(1,M)/XNA
      FT(5,M) = FT(5,M)/XNA
      FT(6,M) = FT(6,M)/XNA
      DO 5300 I=1,NBX+1
      DO 5300 K=1,NBZ+1
      L=((NBZ+1)*(NBY+1)*(I-1)+(K-1))*3
      FT(2,M) = FT(2,M) - FB(L+2,M)
      FT(4,M) = FT(4,M) - FB(L+3,M)
5300 FT(2,M) = FT(2,M)/YNA
      FT(4,M) = FT(4,M)/YNA
      DO 5350 I=1,NBX+1
      DO 5350 J=1,NBY+1
      L=((NBZ+1)*(NBY+1)*(I-1)+(NBZ+1)*(J-1))*3
5350 FT(3,M) = FT(3,M) - FB(L+3,M)
      FT(3,M) = FT(3,M)/ZNA
5400 CONTINUE
      DO 5420 I=1,6
      DO 5420 J=1,6
5420 FTT(I,J) = FT(I,J)
C
C CALCULATE THE ELASTIC CONSTANTS OF THE UNIT CELL
C
      CALL INV(FTT)
      DO 5450 I=1,6
      DO 5450 J=1,6

```

```

5450 FT(I,J)-FTT(I,J)
      EX=1.0/FT(1,1)
      EY=1.0/FT(2,2)
      EZ=1.0/FT(3,3)
      GXY=1.0/FT(6,6)
      GYZ=1.0/FT(4,4)
      GXZ=1.0/FT(5,5)
      PRXY=-FT(2,1)/FT(1,1)
      PRYX=-FT(1,2)/FT(2,2)
      PRXZ=-FT(3,1)/FT(1,1)
      PRZX=-FT(1,3)/FT(3,3)
      PRYZ=-FT(3,2)/FT(2,2)
      PRZY=-FT(2,3)/FT(3,3)
C     MIXXY=FT(4,1)/FT(1,1)
C     MIXXZ=FT(6,1)/FT(1,1)
C     MIXYZ=FT(5,1)/FT(1,1)
C     MIYXY=FT(4,2)/FT(2,2)
C     MIYXZ=FT(6,2)/FT(2,2)
C     MIYYZ=FT(5,2)/FT(2,2)
C     MIZXY=FT(4,3)/FT(3,3)
C     MIZXZ=FT(6,3)/FT(3,3)
C     MIZYZ=FT(5,3)/FT(3,3)
C     CXYXZ=FT(4,5)/FT(5,5)
C     CXYYZ=FT(4,6)/FT(6,6)
C     CXZYZ=FT(5,6)/FT(6,6)
      WRITE(6,9560)
      WRITE(6,9560)
      WRITE(6,9600)
      WRITE(6,9560)
      WRITE(6,9500) EX,EY,EZ
      WRITE(6,9510) GYZ,GXZ,GXY
      WRITE(6,9520) PRYZ,PRXZ,PRXY
      WRITE(6,9525) PRZY,PRZX,PRYX
C     WRITE(6,9530) MIXYZ,MIYYZ,MIZYZ
C     WRITE(6,9540) MIXXZ,MIYXZ,MIZXZ
C     WRITE(6,9550) MIXXY,MIYXY,MIZXY
      WRITE(6,9560)
      WRITE(6,9560)

C
C READ THE APPLIED STRESSES
C
      WRITE(6,9610)
      WRITE(6,9560)
      WRITE(6,9560)
      WRITE(6,9620)
      READ(5,9010) SX
      WRITE(6,9895) SX
      WRITE(6,9630)
      READ(5,9010) SY
      WRITE(6,9895) SY
      WRITE(6,9640)
      READ(5,9010) SZ
      WRITE(6,9895) SZ
      WRITE(6,9650)
      READ(5,9010) SYZ
      WRITE(6,9895) SYZ
      WRITE(6,9660)
      READ(5,9010) SXZ
      WRITE(6,9895) SXZ
      WRITE(6,9670)
      READ(5,9010) SXY
      WRITE(6,9895) SXY
      WRITE(6,9560)
      WRITE(6,9560)

```

```

C
C CALCULATE THE AVERAGE STRAINS

```

```

C
STN(1)=-SX*FT(1,1)+SY*FT(1,2)+SZ*FT(1,3)+SYZ*FT(1,4)
STN(1)=STN(1)+SXZ*FT(1,5)+SXY*FT(1,6)
STN(2)=-SX*FT(2,1)+SY*FT(2,2)+SZ*FT(2,3)+SYZ*FT(2,4)
STN(2)=STN(2)+SXZ*FT(2,5)+SXY*FT(2,6)
STN(3)=-SX*FT(3,1)+SY*FT(3,2)+SZ*FT(3,3)+SYZ*FT(3,4)
STN(3)=STN(3)+SXZ*FT(3,5)+SXY*FT(3,6)
STN(4)=-SX*FT(4,1)+SY*FT(4,2)+SZ*FT(4,3)+SYZ*FT(4,4)
STN(4)=STN(4)+SXZ*FT(4,5)+SXY*FT(4,6)
STN(5)=-SX*FT(5,1)+SY*FT(5,2)+SZ*FT(5,3)+SYZ*FT(5,4)
STN(5)=STN(5)+SXZ*FT(5,5)+SXY*FT(5,6)
STN(6)=-SX*FT(6,1)+SY*FT(6,2)+SZ*FT(6,3)+SYZ*FT(6,4)
STN(6)=STN(6)+SXZ*FT(6,5)+SXY*FT(6,6)

C
C GET NODAL DISPLACEMENTS CORRESPONDING TO THE AVG. STRAINS
C
DO 5500 I=1,NP
VU(I)=STN(1)*UVW(I,1)+STN(2)*UVW(I,2)+STN(3)*UVW(I,3)
VU(I)=VU(I)+STN(4)*UVW(I,4)+STN(5)*UVW(I,5)+STN(6)*UVW(I,6)
5500 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C BEGIN DO LOOP FOR STRESSES IN EACH ELEMENT C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C ESTABLISH ELEMENT DIMENSIONS
C
DO 8000 I=1,NBX
DO 8000 J=1,NBY
DO 8000 K=1,NBZ
A=FDX(I+1)-FDX(I)
A=A*XL/100.
AA=0.5*A
B=FDY(J+1)-FDY(J)
B=B*YL/100.
BB=0.5*B
C=FDZ(K+1)-FDZ(K)
C=C*ZL/100.
CC=0.5*C
DO 5600 III=1,6
DO 5600 JJJ=1,24
5600 BM(III,JJJ)=0.0
X=0.0
Y=0.0
Z=0.0

C
C GET STRAIN / DISPLACEMENT MATRIX
C
CALL GETB(AA,BB,CC,X,Y,Z,BM)

C
C GET CORNER DISPLACEMENTS
C
L=((NBX+1)*(NBZ+1)*(I-1)+(NBZ+1)*(J-1)+(K-1))*3
LL=L+(NBZ+1)*3
LLL=L+(NBZ+1)*(NBX+1)*3
LLLL=LLL+(NBZ+1)*3
DO 5700 M=1,6
UVWS(M)=VU(L+M)
UVWS(M+6)=VU(LL+M)
UVWS(M+12)=VU(LLL+M)
5700 UVWS(M+18)=VU(LLLL+M)

C
C GET AVERAGE ELEMENT STRAINS
C
DO 5800 M=1,6
TS(M)=0.0

```

```

DO 5800 N=1,24
5800 TS(M)=TS(M)+BM(M,N)*UVWS(N)
DUM=TS(4)
TS(4)=TS(5)
TS(5)=TS(6)
TS(6)=DUM
C
C RECALL NUMBER OF JUNCTIONS IN ELEMENT
C
JTC=NJC(I,J,K)
C
C RECALL MATL. NO. AND FIBER DIRECTION ANGLES (ONLY BRANCH)
C
IF(JTC.LT.1) THEN
MN=MN1(I,J,K)
A1=ANG1A(I,J,K)
A2=ANG1B(I,J,K)
C
C GET AVERAGE STRAINS IN MATERIAL COORDINATES
C
CALL TRANS2(A1,A2,T)
DO 5900 M=1,6
SIG(M)=0.0
DO 5900 N=1,6
SS(M,N)=0.0
5900 SIG(M)=SIG(M)+T(M,N)*TS(N)
C WRITE(6,9031) I,J,K
C WRITE(6,9020) (SIG(M),M=1,6)
C
C GET STRESSES IN THE MATERIAL
C
CALL GETSS(MM,MN,PROP,SS)
CALL GETMS(MM,MN,PROP,SIG,SAFE)
DO 6010 M=1,6
TS1(M)=0.0
DO 6010 N=1,6
6010 TS1(M)=TS1(M)+SS(M,N)*SIG(N)
WRITE(6,9560)
WRITE(6,9690) I,J,K
WRITE(6,9692) MN
WRITE(6,9694) TS1(1),TS1(2),TS1(3)
WRITE(6,9696) TS1(4),TS1(5),TS1(6)
WRITE(6,9698) SAFE
WRITE(6,9560)
END IF
C
C RECALL INTERFACIAL NORMAL DIRECTION ANGLES (FIRST BRANCH)
C
IF(JTC.EQ.1) THEN
A1=AGN1A(I,J,K)
A2=AGN1B(I,J,K)
C
C GET AVG. STRAINS IN INTERFACIAL COORDINATES
C
CALL TRANS2(A1,A2,T)
DO 6050 II=1,6
DUM=0.0
DO 6040 JJ=1,6
6040 DUM=DUM+T(II,JJ)*TS(JJ)
6050 TSS(II)=DUM
C
C RECALL MATL.NO. AND FIBER DIRECTION ANGLES (FIRST BRANCH)
C
MN=MN1(I,J,K)
A1=ANG1A(I,J,K)
A2=ANG1B(I,J,K)

```



```

C
C GET STRESS/STRAIN MATRIX IN GLOBAL COORDINATES
C
    CALL GETSS(MM,MN,PROP,SS)
    CALL TRANS2(A1,A2,T)
    DO 6400 II=1,6
    DO 6400 JJ=1,6
    SUM=0.0
    DO 6350 KK=1,6
6350 SUM=SUM+SS(II,KK)*T(KK,JJ)
6400 PR(II,JJ)=SUM
    DO 6450 II=1,6
    DO 6450 JJ=1,6
    SUM=0.0
    DO 6440 KK=1,6
6440 SUM=SUM+T(KK,II)*PR(KK,JJ)
6450 SS1(II,JJ)=SUM
C
C RECALL MATL.NO. AND FIBER DIRECTION ANGLES (SECOND BRANCH)
C
    MN=MN2(I,J,K)
    A1=ANG2A(I,J,K)
    A2=ANG2B(I,J,K)
C
C GET STRESS/STRAIN MATRIX IN GLOBAL COORDINATES
C
    CALL GETSS(MM,MN,PROP,SS)
    CALL TRANS2(A1,A2,T)
    DO 6500 II=1,6
    DO 6500 JJ=1,6
    SUM=0.0
    DO 6490 KK=1,6
6490 SUM=SUM+SS(II,KK)*T(KK,JJ)
6500 PR(II,JJ)=SUM
    DO 6550 II=1,6
    DO 6550 JJ=1,6
    SUM=0.0
    DO 6540 KK=1,6
6540 SUM=SUM+T(KK,II)*PR(KK,JJ)
6550 SS2(II,JJ)=SUM
C
C RECALL FIRST BRANCH VOL.FRACT. AND INTERFACIAL NORMAL ANGLES
C
    V1=FV1(I,J,K)
    V2=1.0-V1
    A1=AGN1A(I,J,K)
    A2=AGN1B(I,J,K)
C
C GET S.S. MATRICES IN INTERFACIAL COORDINATES
C
    CALL TRANS1(A1,A2,T)
    DO 6600 II=1,6
    DO 6600 JJ=1,6
    SUM=0.0
    TUM=0.0
    DO 6590 KK=1,6
    SUM=SUM+SS1(II,KK)*T(KK,JJ)
6590 TUM=TUM+SS2(II,KK)*T(KK,JJ)
    RP(II,JJ)=SUM
6600 PR(II,JJ)=TUM
    DO 6650 II=1,6
    DO 6650 JJ=1,6
    SUM=0.0
    TUM=0.0
    DO 6640 KK=1,6
    SUM=SUM+T(KK,II)*RP(KK,JJ)

```

```

6640 TUM=TUM+T(KK,II)*PR(KK,JJ)
      SSS1(II,JJ)=SUM
6650 SSS2(II,JJ)=TUM
C
C DO REPLACEMENT MATERIAL ANALYSIS AT FIRST JUNCTION
C
      CALL GETDD(SSS1,SSS2,V1,V2,DD1,DD2)
C
C GET CONSTITUENT STRAINS IN INTERFACIAL COORDINATES
C
      DO 6711 II=1,6
        TS1(II)=0.0
        TS2(II)=0.0
        DO 6711 JJ=1,6
          TS1(II)=TS1(II)+DD1(II,JJ)*TSS(JJ)
6711 TS2(II)=TS2(II)+DD2(II,JJ)*TSS(JJ)
C
C GET CONSTITUENT STRAINS IN GLOBAL COORDINATES
C
      DO 6720 II=1,6
        DUM=0.0
        TUM=0.0
        DO 6715 JJ=1,6
          DUM=DUM+T(II,JJ)*TS1(JJ)
6715 TUM=TUM+T(II,JJ)*TS2(JJ)
          TS(II)=DUM
6720 TSS(II)=TUM
C
C GET CONSTITUENT STRAINS IN MATERIAL COORDINATES
C
      A1=ANG1A(I,J,K)
      A2=ANG1B(I,J,K)
      CALL TRANS2(A1,A2,T)
      DO 6730 II=1,6
        DUM=0.0
        DO 6725 JJ=1,6
          DUM=DUM+T(II,JJ)*TS(JJ)
6725 TS1(II)=DUM
6730 A1=ANG2A(I,J,K)
          A2=ANG2B(I,J,K)
          CALL TRANS2(A1,A2,T)
          DO 6740 II=1,6
            DUM=0.0
            DO 6735 JJ=1,6
              DUM=DUM+T(II,JJ)*TSS(JJ)
6735 TS2(II)=DUM
6740
C
C GET CONSTITUENT STRESSES IN MATERIAL COORDINATES
C
      MN=MN1(I,J,K)
      CALL GETSS(MM,MN,PROP,SS)
      CALL GETMS(MM,MN,PROP,TS1,SAFE)
      DO 6750 II=1,6
        ST1(II)=0.0
        DO 6750 JJ=1,6
          ST1(II)=ST1(II)+SS(II,JJ)*TS1(JJ)
6750 WRITE(6,9560)
      WRITE(6,9690) I,J,K
      WRITE(6,9692) MN
      WRITE(6,9694) ST1(1),ST1(2),ST1(3)
      WRITE(6,9696) ST1(4),ST1(5),ST1(6)
      WRITE(6,9698) SAFE
      WRITE(6,9560)
      MN=MN2(I,J,K)
      CALL GETSS(MM,MN,PROP,SS)
      CALL GETMS(MM,MN,PROP,TS2,SAFE)

```

```

DO 6760 II=1,6
ST2(II)=0.0
DO 6760 JJ=1,6
6760 ST2(II)=ST2(II)+SS(II,JJ)*TS2(JJ)
WRITE(6,9560)
WRITE(6,9690) I,J,K
WRITE(6,9692) MN
WRITE(6,9694) ST2(1),ST2(2),ST2(3)
WRITE(6,9696) ST2(4),ST2(5),ST2(6)
WRITE(6,9698) SAFE
WRITE(6,9560)
END IF

C
IF(JTC.EQ.2) THEN
C
C RECALL MATL.NO. AND FIBER DIRECTION ANGLES (FIRST BRANCH)
C
MN=MN1(I,J,K)
A1=ANG1A(I,J,K)
A2=ANG1B(I,J,K)

C
C GET STRESS/STRAIN MATRIX IN GLOBAL COORDINATES
C
CALL GETSS(MM,MN,PROP,SS)
CALL TRANS2(A1,A2,T)
DO 7040 II=1,6
DO 7040 JJ=1,6
SUM=0.0
DO 7035 KK=1,6
7035 SUM=SUM+SS(II,KK)*T(KK,JJ)
7040 PR(II,JJ)=SUM
DO 7045 II=1,6
DO 7045 JJ=1,6
SUM=0.0
DO 7044 KK=1,6
7044 SUM=SUM+T(KK,II)*PR(KK,JJ)
7045 SS1(II,JJ)=SUM

C
C RECALL MATL.NO. AND FIBER DIRECTION ANGLES (SECOND BRANCH)
C
MN=MN2(I,J,K)
A1=ANG2A(I,J,K)
A2=ANG2B(I,J,K)

C
C GET STRESS/STRAIN MATRIX IN GLOBAL COORDINATES
C
CALL GETSS(MM,MN,PROP,SS)
CALL TRANS2(A1,A2,T)
DO 7050 II=1,6
DO 7050 JJ=1,6
SUM=0.0
DO 7049 KK=1,6
7049 SUM=SUM+SS(II,KK)*T(KK,JJ)
7050 PR(II,JJ)=SUM
DO 7055 II=1,6
DO 7055 JJ=1,6
SUM=0.0
DO 7054 KK=1,6
7054 SUM=SUM+T(KK,II)*PR(KK,JJ)
7055 SS2(II,JJ)=SUM

C
C RECALL FIRST BRANCH VOL.FRACT. AND INTERFACIAL NORMAL ANGLES
C
V1=FV1(I,J,K)
V2=1.0-V1
A1=AGN1A(I,J,K)

```

```

      A2=AGN1B(I,J,K)
C
C GET S.S. MATRICES IN INTERFACIAL COORDINATES
C
      CALL TRANS1(A1,A2,T)
      DO 7060 II=1,6
      DO 7060 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 7059 KK=1,6
      SUM=SUM+SS1(II, KK)*T(KK, JJ)
7059 TUM=TUM+SS2(II, KK)*T(KK, JJ)
      RP(II, JJ)=SUM
7060 PR(II, JJ)=TUM
      DO 7065 II=1,6
      DO 7065 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 7064 KK=1,6
      SUM=SUM+T(KK, II)*RP(KK, JJ)
7064 TUM=TUM+T(KK, II)*PR(KK, JJ)
      SSS1(II, JJ)=SUM
7065 SSS2(II, JJ)=TUM
C
C DO REPLACEMENT MATERIAL ANALYSIS AT FIRST JUNCTION
C
      CALL GETDD(SSS1, SSS2, V1, V2, DD1, DD2)
      DO 7075 II=1,6
      DO 7075 JJ=1,6
      DUM=0.0
      DO 7074 KK=1,6
7074 DUM=DUM+V1*SSS1(II, KK)*DD1(KK, JJ)+V2*SSS2(II, KK)*DD2(KK, JJ)
7075 DD(II, JJ)=DUM
C
C GET REPLACEMENT S.S. MATRIX IN GLOBAL COORDINATES
C
      CALL TRANS2(A1,A2,T)
      DO 7079 II=1,6
      DO 7079 JJ=1,6
      SUM=0.0
      DO 7078 KK=1,6
7078 SUM=SUM+DD(II, KK)*T(KK, JJ)
7079 PR(II, JJ)=SUM
      DO 7085 II=1,6
      DO 7085 JJ=1,6
      SUM=0.0
      DO 7084 KK=1,6
7084 SUM=SUM+T(KK, II)*PR(KK, JJ)
7085 SS1(II, JJ)=SUM
C
C RECALL MATL.NO. AND FIBER DIRECTION ANGLES (THIRD BRANCH)
C
      MN=MN3(I,J,K)
      A1=ANG3A(I,J,K)
      A2=ANG3B(I,J,K)
C
C GET STRESS/STRAIN MATRIX IN GLOBAL COORDINATES
C
      CALL GETSS(MM,MN,PROP,SS)
      CALL TRANS2(A1,A2,T)
      DO 7150 II=1,6
      DO 7150 JJ=1,6
      SUM=0.0
      DO 7149 KK=1,6
7149 SUM=SUM+SS(II, KK)*T(KK, JJ)
7150 PR(II, JJ)=SUM

```

```

      DO 7155 II=1,6
      DO 7155 JJ=1,6
      SUM=0.0
      DO 7154 KK=1,6
7154 SUM=SUM+T(KK,II)*PR(KK,JJ)
7155 SSS2(II,JJ)=SUM
C
C RECALL SECOND BRANCH VOL.FRACT. AND INTERFACIAL NORMAL ANGLES
C
      V2=FV2(I,J,K)
      V1=1.0-V1
      A1=AGN2A(I,J,K)
      A2=AGN2B(I,J,K)
C
C GET S.S. MATRICES IN INTERFACIAL COORDINATES
C
      CALL TRANS1(A1,A2,T)
      DO 7160 II=1,6
      DO 7160 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 7159 KK=1,6
      SUM=SUM+SS1(II,KK)*T(KK,JJ)
7159 TUM=TUM+SS2(II,KK)*T(KK,JJ)
      RP(II,JJ)=SUM
7160 PR(II,JJ)=TUM
      DO 7165 II=1,6
      DO 7165 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 7164 KK=1,6
      SUM=SUM+T(KK,II)*RP(KK,JJ)
7164 TUM=TUM+T(KK,II)*PR(KK,JJ)
      SSS1(II,JJ)=SUM
7165 SSS2(II,JJ)=TUM
C
C DO REPLACEMENT MATERIAL ANALYSIS AT SECOND JUNCTION
C
      CALL GETDD(SSS1,SSS2,V1,V2,DD1,DD2)
C
C GET AVERAGE STRAINS IN INTERFACIAL COORDINATES
C
      A1=AGN2A(I,J,K)
      A2=AGN2B(I,J,K)
      CALL TRANS2(A1,A2,T)
      DO 7250 II=1,6
      DUM=0.0
      DO 7240 JJ=1,6
7240 DUM=DUM+T(II,JJ)*TS(JJ)
7250 TSS(II)=DUM
C
C GET CONSTITUENT STRAINS IN INTERFACIAL COORDINATES
C
      DO 7311 II=1,6
      TS1(II)=0.0
      TS2(II)=0.0
      DO 7311 JJ=1,6
      TS1(II)=TS1(II)+DD1(II,JJ)*TSS(JJ)
7311 TS2(II)=TS2(II)+DD2(II,JJ)*TSS(JJ)
C
C GET CONSTITUENT STRAINS IN GLOBAL COORDINATES
C
      CALL TRANS1(A1,A2,T)
      DO 7320 II=1,6
      DUM=0.0
      TUM=0.0

```

```

DO 7315 JJ=1,6
DUM=DUM+T(II,JJ)*TS1(JJ)
7315 TUM=TUM+T(II,JJ)*T2(JJ)
TS(II)=DUM
7320 TSS(II)=TUM
A1=ANG3A(I,J,K)
A2=ANG3B(I,J,K)
CALL TRANS2(A1,A2,T)
DO 7330 II=1,6
DUM=0.0
DO 7325 JJ=1,6
7315 DUM=DUM+T(II,JJ)*TSS(JJ)
7330 TS2(II)=DUM
C
C GET CONSTITUENT STRESSES IN MATERIAL COORDINATES
C
MN=MN3(I,J,K)
CALL GETSS(MM,MN,PROP,SS)
CALL GETMS(MM,MN,PROP,TS2,SAFE)
DO 7350 II=1,6
ST2(II)=0.0
DO 7350 JJ=1,6
7350 ST2(II)=ST2(II)+SS(II,JJ)*TS2(JJ)
WRITE(6,9560)
WRITE(6,9690) I,J,K
WRITE(6,9692) MN
WRITE(6,9694) ST2(1),ST2(2),ST2(3)
WRITE(6,9696) ST2(4),ST2(5),ST2(6)
WRITE(6,9698) SAFE
WRITE(6,9560)
C
C RECALL FIRST BRANCH INTERFACIAL NORMAL ANGLES
C
A1=AGN1A(I,J,K)
A2=AGN1B(I,J,K)
C
C GET AVERAGE STRAINS IN INTERFACIAL COORDINATES
C
CALL TRANS2(A1,A2,T)
DO 7370 II=1,6
DUM=0.0
DO 7360 JJ=1,6
7360 DUM=DUM+T(II,JJ)*TS(JJ)
7370 TSS(II)=DUM
C
C RECALL FIRST BRANCH MATL. NUMBER AND FIBER ANGLES
C
MN=MN1(I,J,K)
A1=ANG1A(I,J,K)
A2=ANG1B(I,J,K)
C
C GET FIRST BRANCH S.S. MATRIX IN GLOBAL COORDINATES
C
CALL GETSS(MM,MN,PROP,SS)
CALL TRANS2(A1,A2,T)
DO 7400 II=1,6
DO 7400 JJ=1,6
SUM=0.0
DO 7390 KK=1,6
7390 SUM=SUM+SS(II,KK)*T(KK,JJ)
7400 PR(II,JJ)=SUM
DO 7450 II=1,6
DO 7450 JJ=1,6
SUM=0.0
DO 7440 KK=1,6
7440 SUM=SUM+T(KK,II)*PR(KK,JJ)

```

```

7450 SS1(II,JJ)=SUM
C
C RECALL SECOND BRANCH MATL. NUMBER AND FIBER ANGLES
C
      MN=MN2(I,J,K)
      A1=ANG2A(I,J,K)
      A2=ANG2B(I,J,K)
C
C GET SECOND BRANCH S.S. MATRIX IN GLOBAL COORDINATES
C
      CALL GETSS(MM,MN,PROP,SS)
      CALL TRANS2(A1,A2,T)
      DO 7500 II=1,6
      DO 7500 JJ=1,6
      SUM=0.0
      DO 7490 KK=1,6
7490 SUM=SUM+SS(II,KK)*T(KK,JJ)
7500 PR(II,JJ)=SUM
      DO 7550 II=1,6
      DO 7550 JJ=1,6
      SUM=0.0
      DO 7540 KK=1,6
7540 SUM=SUM+T(KK,II)*PR(KK,JJ)
7550 SS2(II,JJ)=SUM
C
C RECALL FIRST BRANCH VOL. FRACT. AND INTER. NORMAL ANGLES
C
      V1=FV1(I,J,K)
      V2=1.0-V1
      A1=AGN1A(I,J,K)
      A2=AGN1B(I,J,K)
C
C GET MATL S.S. MATRICES IN INTERFACIAL COORDINATES
C
      CALL TRANS1(A1,A2,T)
      DO 7600 II=1,6
      DO 7600 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 7590 KK=1,6
      SUM=SUM+SS1(II,KK)*T(KK,JJ)
7590 TUM=TUM+SS2(II,KK)*T(KK,JJ)
      RP(II,JJ)=SUM
7600 PR(II,JJ)=TUM
      DO 7650 II=1,6
      DO 7650 JJ=1,6
      SUM=0.0
      TUM=0.0
      DO 7640 KK=1,6
      SUM=SUM+T(KK,II)*RP(KK,JJ)
7640 TUM=TUM+T(KK,II)*PR(KK,JJ)
      SSS1(II,JJ)=SUM
7650 SSS2(II,JJ)=TUM
C
C DO REPLACEMENT MATERIAL ANALYSIS
C
      CALL GETDD(SSS1,SSS2,V1,V2,DD1,DD2)
C
C GET CONSTITUENT STRAINS IN INTERFACIAL COORDINATES
C
      DO 7711 II=1,6
      TS1(II)=0.0
      TS2(II)=0.0
      DO 7711 JJ=1,6
      TS1(II)=TS1(II)+DD1(II,JJ)*TSS(JJ)
7711 TS2(II)=TS2(II)+DD2(II,JJ)*TSS(JJ)

```

```

C
C GET CONSTITUENT STRAINS IN GLOBAL COORDINATES
C
    DO 7720 II=1,6
    DUM=0.0
    TUM=0.0
    DO 7715 JJ=1,6
    DUM=DUM+T(II,JJ)*TS1(JJ)
7715 TUM=TUM+T(II,JJ)*TS2(JJ)
    TS(II)=DUM
7720 TSS(II)=TUM
C
C RECALL FIRST BRANCH FIBER ANGLES
C
    A1=ANG1A(I,J,K)
    A2=ANG1B(I,J,K)
C
C GET FIRST BRANCH STRAINS IN MATERIAL COORDINATES
C
    CALL TRANS2(A1,A2,T)
    DO 7730 II=1,6
    DUM=0.0
    DO 7725 JJ=1,6
7725 DUM=DUM+T(II,JJ)*TS(JJ)
7730 TS1(II)=DUM
C
C RECALL SECOND BRANCH FIBER ANGLES
C
    A1=ANG2A(I,J,K)
    A2=ANG2B(I,J,K)
C
C GET SECOND BRANCH STRAINS IN MATERIAL COORDINATES
C
    CALL TRANS2(A1,A2,T)
    DO 7740 II=1,6
    DUM=0.0
    DO 7735 JJ=1,6
7735 DUM=DUM+T(II,JJ)*TSS(JJ)
7740 TS2(II)=DUM
C
C GET FIRST BRANCH STRESSES
C
    MN=MN1(I,J,K)
    CALL GETSS(MM,MN,PROP,SS)
    CALL GETMS(MM,MN,PROP,TS1,SAFE)
    DO 7750 II=1,6
    ST1(II)=0.0
    DO 7750 JJ=1,6
7750 ST1(II)=ST1(II)+SS(II,JJ)*TS1(JJ)
    WRITE(6,9560)
    WRITE(6,9690) I,J,K
    WRITE(6,9692) MN
    WRITE(6,9694) ST1(1),ST1(2),ST1(3)
    WRITE(6,9696) ST1(4),ST1(5),ST1(6)
    WRITE(6,9698) SAFE
    WRITE(6,9560)
C
C GET SECOND BRANCH STRESSES
C
    MN=MN2(I,J,K)
    CALL GETSS(MM,MN,PROP,SS)
    CALL GETMS(MM,MN,PROP,TS2,SAFE)
    DO 7760 II=1,6
    ST2(II)=0.0
    DO 7760 JJ=1,6
7760 ST2(II)=ST2(II)+SS(II,JJ)*TS2(JJ)

```



```

WRITE(6,9560)
WRITE(6,9690) I,J,K
WRITE(6,9692) MN
WRITE(6,9694) ST2(1),ST2(2),ST2(3)
WRITE(6,9696) ST2(4),ST2(5),ST2(6)
WRITE(6,9698) SAFE
WRITE(6,9560)
END IF
8000 CONTINUE
C
9000 FORMAT(F12.5)
9010 FORMAT(F12.2)
9015 FORMAT(E16.6)
9020 FORMAT(6F12.2)
9025 FORMAT(10X,4E14.4)
9030 FORMAT(I5)
C9031 FORMAT(3I5)
9060 FORMAT(1H,'INPUT NUMBER OF JUNCTIONS AT LOCATION',3I4)
9080 FORMAT(1H,'INPUT NO. SUBCELLS (X DIR.) IN UNIT CELL')
9090 FORMAT(1H,'INPUT NO. SUBCELLS (Y DIR.) IN UNIT CELL')
9095 FORMAT(1H,'INPUT NO. SUBCELLS (Z DIR.) IN UNIT CELL')
9100 FORMAT(1H,'INPUT NO. COMPOSITE MATERIALS NEEDED,NM')
9120 FORMAT(1H,'INPUT E IN FIBER DIRECTION')
9130 FORMAT(1H,'INPUT E NORMAL TO FIBER DIRECTION')
9140 FORMAT(1H,'INPUT MAJOR POISSONS RATIO IN LT PLANE')
9150 FORMAT(1H,'INPUT POISSONS RATIO IN TT PLANE')
9160 FORMAT(1H,'INPUT SHEAR MODULUS G IN LT PLANE')
9170 FORMAT(1H,'INPUT SHEAR MODULUS G IN TT PLANE')
9175 FORMAT(1H,'INPUT LONG.TENSION ALLOWABLE')
9177 FORMAT(1H,'INPUT LONG COMPRESSION ALLOWABLE')
9178 FORMAT(1H,'INPUT TRANS. TENSION ALLOWABLE')
9179 FORMAT(1H,'INPUT TRANS. COMPRESSION ALLOWABLE')
9180 FORMAT(1H,'SELECT A MATERIAL NUMBER FROM ONE TO TEN')
9190 FORMAT(1H,'MATERIAL PROPERTY DATA ECHO')
9320 FORMAT(1H,'SPECIFY THE CURRENT MATL. ID. NO.')
9440 FORMAT(1H,'INPUT SIDE LENGTH OF UNIT CELL IN X DIR.')
9450 FORMAT(1H,'INPUT SIDE LENGTH OF UNIT CELL IN Y DIR.')
9455 FORMAT(1H,'INPUT SIDE LENGTH OF UNIT CELL IN Z DIR.')
9460 FORMAT(1H,'INPUT DIST.(%) ORIGIN TO UNIT CELL NODE',I3)
9480 FORMAT(1H,'INPUT 1ST FIBER SPHERICAL ANGLE')
9485 FORMAT(1H,'INPUT 1ST INTERFACIAL NORMAL ANGLE')
9490 FORMAT(1H,'INPUT 2ND FIBER SPHERICAL ANGLE')
9495 FORMAT(1H,'INPUT 2ND INTERFACIAL NORMAL ANGLE')
9500 FORMAT(1H,'EX,EY,EZ = ',3F12.2)
9510 FORMAT(1H,'GYZ,GXZ,GXY = ',3F12.2)
9520 FORMAT(1H,'MUYZ,MUXZ,MUXY = ',3F12.4)
9525 FORMAT(1H,'MUZY,MUZX,MUYX = ',3F12.4)
9530 FORMAT(1H,'NUYZ,X ; NUYZ,Y ; NUYZ,Z = ',3F12.4)
9540 FORMAT(1H,'NUXZ,X ; NUXZ,Y ; NUXZ,Z = ',3F12.4)
9550 FORMAT(1H,'NUXY,X ; NUXY,Y ; NUXY,Z = ',3F12.4)
9560 FORMAT(1H)
9600 FORMAT(1H,13X,'ELASTIC CONSTANTS OF THE COMPOSITE ')
9610 FORMAT(1H,13X,'INPUT APPLIED STRESSES IN X,Y,Z COORDINATES')
9620 FORMAT(1H,5X,'INPUT X NORMAL STRESS ')
9630 FORMAT(1H,5X,'INPUT Y NORMAL STRESS ')
9640 FORMAT(1H,5X,'INPUT Z NORMAL STRESS ')
9650 FORMAT(1H,5X,'INPUT YZ SHEAR STRESS ')
9660 FORMAT(1H,5X,'INPUT XZ SHEAR STRESS ')
9670 FORMAT(1H,5X,'INPUT XY SHEAR STRESS ')
9690 FORMAT(1H,5X,'STRESSES IN ELEMENT NO.',3I3)
9692 FORMAT(1H,'MATERIAL NO.',I3)
9694 FORMAT(1H,'NORMAL 1,2,3 ',3F14.2)
9696 FORMAT(1H,'SHEAR 23,13,12 ',3F14.2)
9698 FORMAT(1H,'MINIMUM MARGIN OF SAFETY IS ',F12.4)
9700 FORMAT(1H,'INPUT MATL. NO. 1 AT ',3I4)
9710 FORMAT(1H,'INPUT MATL. NO. 2 AT ',3I4)

```

```

9715 FORMAT(1H , 'INPUT MATL. NO. 3 AT ',3I4)
9720 FORMAT(1H , 'INPUT 1ST MATL. VOLUME FRACTION')
9895 FORMAT(1H , '      ((' ,F7.1,')'))
9898 FORMAT(1H , '      ((' ,F4.1,')'))
9899 FORMAT(1H , '      ((' ,I3,')'))
9999 END

```

C-----
C MATRIX INVERSION BY PARTITIONING OF A 6X6 MATRIX
C

```

SUBROUTINE INV(A)
  DIMENSION A(6,6),B(4,4),X(4,4),Y(4,4),Z(4,4)
  D=A(1,1)*(A(2,2)*A(3,3)-A(2,3)*A(3,2))
  1 -A(2,1)*(A(1,2)*A(3,3)-A(1,3)*A(3,2))
  2 +A(3,1)*(A(1,2)*A(2,3)-A(2,2)*A(1,3))
  IF(D.EQ.0.0) GOTO 700
  B(1,1)= (A(2,2)*A(3,3)-A(2,3)*A(3,2))/D
  B(1,2)= -(A(1,2)*A(3,3)-A(1,3)*A(3,2))/D
  B(1,3)= (A(1,2)*A(2,3)-A(1,3)*A(2,2))/D
  B(2,1)= -(A(2,1)*A(3,3)-A(2,3)*A(3,1))/D
  B(2,2)= (A(1,1)*A(3,3)-A(1,3)*A(3,1))/D
  B(2,3)= -(A(1,1)*A(2,3)-A(1,3)*A(2,1))/D
  B(3,1)= (A(2,1)*A(3,2)-A(2,2)*A(3,1))/D
  B(3,2)= -(A(1,1)*A(3,2)-A(1,2)*A(3,1))/D
  B(3,3)= (A(1,1)*A(2,2)-A(1,2)*A(2,1))/D
  DO 100 I=1,3
  DO 100 J=1,3
  X(I,J)=0.0
  Y(I,J)=0.0
100 Z(I,J)=A(I+3,J+3)
  DO 200 I=1,3
  DO 200 J=1,3
  DO 200 K=1,3
  X(I,J)=X(I,J)+B(I,K)*A(K,J+3)
200 Y(I,J)=Y(I,J)+A(I+3,K)*B(K,J)
  DO 300 I=1,3
  DO 300 J=1,3
  DO 300 K=1,3
300 Z(I,J)=Z(I,J)-Y(I,K)*A(K,J+3)
  D=Z(1,1)*(Z(2,2)*Z(3,3)-Z(2,3)*Z(3,2))
  1 -Z(2,1)*(Z(1,2)*Z(3,3)-Z(1,3)*Z(3,2))
  2 +Z(3,1)*(Z(1,2)*Z(2,3)-Z(2,2)*Z(1,3))
  IF(D.EQ.0.0) GOTO 700
  DO 400 I=1,6
  DO 400 J=1,6
400 A(I,J)=0.0
  A(4,4)= (Z(2,2)*Z(3,3)-Z(2,3)*Z(3,2))/D
  A(4,5)= -(Z(1,2)*Z(3,3)-Z(1,3)*Z(3,2))/D
  A(4,6)= (Z(1,2)*Z(2,3)-Z(1,3)*Z(2,2))/D
  A(5,4)= -(Z(2,1)*Z(3,3)-Z(2,3)*Z(3,1))/D
  A(5,5)= (Z(1,1)*Z(3,3)-Z(1,3)*Z(3,1))/D
  A(5,6)= -(Z(1,1)*Z(2,3)-Z(1,3)*Z(2,1))/D
  A(6,4)= (Z(2,1)*Z(3,2)-Z(2,2)*Z(3,1))/D
  A(6,5)= -(Z(1,1)*Z(3,2)-Z(1,2)*Z(3,1))/D
  A(6,6)= (Z(1,1)*Z(2,2)-Z(1,2)*Z(2,1))/D
  DO 500 I=1,3
  DO 500 J=1,3
  A(I,J)=B(I,J)
  DO 500 K=1,3
  A(I,J+3)=A(I,J+3)-X(I,K)*A(K+3,J+3)
500 A(I+3,J)=A(I+3,J)-A(I+3,K+3)*Y(K,J)
  DO 600 I=1,3
  DO 600 J=1,3
  DO 600 K=1,3
600 A(I,J)=A(I,J)-A(I,K+3)*Y(K,J)
  GO TO 2000
700 D= A(1,1)*(A(2,2)*(A(3,3)*A(4,4)-A(3,4)*A(4,3))

```

```

1      -A(2,3)*(A(3,2)*A(4,4)-A(3,4)*A(4,2))
2      +A(2,4)*(A(3,2)*A(4,3)-A(3,3)*A(4,2))
D=D-A(1,2)*(A(2,1)*(A(3,3)*A(4,4)-A(3,4)*A(4,3))
1      -A(2,3)*(A(3,1)*A(4,4)-A(3,4)*A(4,1))
2      +A(2,4)*(A(3,1)*A(4,3)-A(3,3)*A(4,1))
D=D+A(1,3)*(A(2,1)*(A(3,2)*A(4,4)-A(3,4)*A(4,2))
1      -A(2,2)*(A(3,1)*A(4,4)-A(3,4)*A(4,1))
2      +A(2,4)*(A(3,1)*A(4,2)-A(3,2)*A(4,1))
D=D-A(1,4)*(A(2,1)*(A(3,2)*A(4,3)-A(3,3)*A(4,2))
1      -A(2,2)*(A(3,1)*A(4,3)-A(3,3)*A(4,1))
2      +A(2,3)*(A(3,1)*A(4,2)-A(3,2)*A(4,1))
C      WRITE(6,9010) D
C9010  FORMAT(F12.2)
      IF(D.EQ.0.0) D=1.0
B(1,1)= +A(2,2)*(A(3,3)*A(4,4)-A(3,4)*A(4,3))
1      -A(2,3)*(A(3,2)*A(4,4)-A(3,4)*A(4,2))
2      +A(2,4)*(A(3,2)*A(4,3)-A(3,3)*A(4,2))
B(1,2)= -A(1,2)*(A(3,3)*A(4,4)-A(3,4)*A(4,3))
1      +A(1,3)*(A(3,2)*A(4,4)-A(3,4)*A(4,2))
2      -A(1,4)*(A(3,2)*A(4,3)-A(3,3)*A(4,2))
B(1,3)= +A(1,2)*(A(2,3)*A(4,4)-A(2,4)*A(4,3))
1      -A(1,3)*(A(2,2)*A(4,4)-A(2,4)*A(4,2))
2      +A(1,4)*(A(2,2)*A(4,3)-A(2,3)*A(4,2))
B(1,4)= -A(1,2)*(A(2,3)*A(3,4)-A(2,4)*A(3,3))
1      +A(1,3)*(A(2,2)*A(3,4)-A(2,4)*A(3,2))
2      -A(1,4)*(A(2,2)*A(3,3)-A(2,3)*A(3,2))
B(2,1)= -A(2,1)*(A(3,3)*A(4,4)-A(3,4)*A(4,3))
1      +A(2,3)*(A(3,1)*A(4,4)-A(3,4)*A(4,1))
2      -A(2,4)*(A(3,1)*A(4,3)-A(3,3)*A(4,1))
B(2,2)= +A(1,1)*(A(3,3)*A(4,4)-A(3,4)*A(4,3))
1      -A(1,3)*(A(3,1)*A(4,4)-A(3,4)*A(4,1))
2      +A(1,4)*(A(3,1)*A(4,3)-A(3,3)*A(4,1))
B(2,3)= -A(1,1)*(A(2,3)*A(4,4)-A(2,4)*A(4,3))
1      +A(1,3)*(A(2,1)*A(4,4)-A(2,4)*A(4,1))
2      -A(1,4)*(A(2,1)*A(4,3)-A(2,3)*A(4,1))
B(2,4)= +A(1,1)*(A(2,3)*A(3,4)-A(2,4)*A(3,3))
1      -A(1,3)*(A(2,1)*A(3,4)-A(2,4)*A(3,1))
2      +A(1,4)*(A(2,1)*A(3,3)-A(2,3)*A(3,1))
B(3,1)= +A(2,1)*(A(3,2)*A(4,4)-A(3,4)*A(4,2))
1      -A(2,2)*(A(3,1)*A(4,4)-A(3,4)*A(4,1))
2      +A(2,4)*(A(3,1)*A(4,2)-A(3,2)*A(4,1))
B(3,2)= -A(1,1)*(A(3,2)*A(4,4)-A(3,4)*A(4,2))
1      +A(1,2)*(A(3,1)*A(4,4)-A(3,4)*A(4,1))
2      -A(1,4)*(A(3,1)*A(4,2)-A(3,2)*A(4,1))
B(3,3)= +A(1,1)*(A(2,2)*A(4,4)-A(2,4)*A(4,2))
1      -A(1,2)*(A(2,1)*A(4,4)-A(2,4)*A(4,1))
2      +A(1,4)*(A(2,1)*A(4,2)-A(2,2)*A(4,1))
B(3,4)= -A(1,1)*(A(2,2)*A(3,4)-A(2,4)*A(3,2))
1      +A(1,2)*(A(2,1)*A(3,4)-A(2,4)*A(3,1))
2      -A(1,4)*(A(2,1)*A(3,2)-A(2,2)*A(3,1))
B(4,1)= -A(2,1)*(A(3,2)*A(4,3)-A(3,3)*A(4,2))
1      +A(2,2)*(A(3,1)*A(4,3)-A(3,3)*A(4,1))
2      -A(2,3)*(A(3,1)*A(4,2)-A(3,2)*A(4,1))
B(4,2)= +A(1,1)*(A(3,2)*A(4,3)-A(3,3)*A(4,2))
1      -A(1,2)*(A(3,1)*A(4,3)-A(3,3)*A(4,1))
2      +A(1,3)*(A(3,1)*A(4,2)-A(3,2)*A(4,1))
B(4,3)= -A(1,1)*(A(2,2)*A(4,3)-A(2,3)*A(4,2))
1      +A(1,2)*(A(2,1)*A(4,3)-A(2,3)*A(4,1))
2      -A(1,3)*(A(2,1)*A(4,2)-A(2,2)*A(4,1))
B(4,4)= +A(1,1)*(A(2,2)*A(3,3)-A(2,3)*A(3,2))
1      -A(1,2)*(A(2,1)*A(3,3)-A(2,3)*A(3,1))
2      +A(1,3)*(A(2,1)*A(3,2)-A(2,2)*A(3,1))
DO 1090 I=1,4
DO 1090 J=1,4
1090 B(I,J)=B(I,J)/D
DO 1100 I=1,4

```

```

DO 1100 J=1,4
X(I,J)=0.0
1100 Y(I,J)=0.0
DO 1110 I=1,2
DO 1110 J=1,2
1110 Z(I,J)=A(I+4,J+4)
DO 1200 I=1,4
DO 1200 J=1,2
DO 1200 K=1,4
1200 X(I,J)=X(I,J)+B(I,K)*A(K,J+4)
DO 1210 I=1,2
DO 1210 J=1,4
DO 1210 K=1,4
1210 Y(I,J)=Y(I,J)+A(I+4,K)*B(K,J)
DO 1300 I=1,2
DO 1300 J=1,2
DO 1300 K=1,4
1300 Z(I,J)=Z(I,J)-Y(I,K)*A(K,J+4)
D=Z(1,1)*Z(2,2)-Z(1,2)*Z(2,1)
IF(D.EQ.0.0) D=1.0
DO 1400 I=1,6
DO 1400 J=1,6
1400 A(I,J)=0.0
A(5,5)=Z(2,2)/D
A(5,6)=-Z(1,2)/D
A(6,5)=-Z(2,1)/D
A(6,6)=Z(1,1)/D
DO 1450 I=1,4
DO 1450 J=1,4
1450 A(I,J)=B(I,J)
DO 1500 I=1,4
DO 1500 J=1,2
DO 1500 K=1,2
1500 A(I,J+4)=A(I,J+4)-X(I,K)*A(K+4,J+4)
DO 1550 I=1,2
DO 1550 J=1,4
DO 1550 K=1,2
1550 A(I+4,J)=A(I+4,J)-A(I+4,K+4)*Y(K,J)
DO 1600 I=1,4
DO 1600 J=1,4
DO 1600 K=1,2
1600 A(I,J)=A(I,J)-A(I,K+4)*Y(K,J)
2000 CONTINUE
RETURN
END

```

```

C-----
SUBROUTINE MATINV (A,NMAX,N,B,MAX,M,DETERM)
C IMPLICIT REAL*8 (A-H,O-Z)
C****
C**** STANDARD MATRIX INVERSION SUBPROGRAM
C****
DIMENSION A(NMAX,NMAX),B(NMAX,MAX)
DIMENSION IPIVOT(300),INDEX(300,2),PIVOT(300)
C
C INITIALIZATION
C
10 DETERM = 1.0
15 DO 20 J=1,N
20 IPIVOT(J) = 0
30 DO 550 I=1,N
C
C SEARCH FOR THE PIVOT ELEMENT
C
40 AMAX = 0.0
45 DO 105 J=1,N
50 IF (IPIVOT(J) .EQ. 1) GOTO 105

```

```

60      DO 100 K=1,N
70      IF (IPIVOT(K) - 1)80,100,740
80      IF (ABS(AMAX) .GE. ABS(A(J,K))) GOTO 100
85      IROW = J
90      ICOLUM = K
95      AMAX = A(J,K)
100     CONTINUE
105     CONTINUE
110     IPIVOT(ICOLUM) = IPIVOT(ICOLUM) + 1
C
C     INTERCHANGE ROWS TO PUT ELEMENT ON DIAGONAL
C
130     IF (IROW .EQ. ICOLUM) GOTO 260
140     DETERM = -DETERM
150     DO 200 L=1,N
160     SWAP = A(IROW,L)
170     A(IROW,L) = A(ICOLUM,L)
200     A(ICOLUM,L) = SWAP
205     IF (M .LE. 0) GOTO 260
210     DO 250 L=1,M
220     SWAP = B(IROW,L)
230     B(IROW,L) = B(ICOLUM,L)
250     B(ICOLUM,L) = SWAP
260     INDEX(I,1) = IROW
270     INDEX(I,2) = ICOLUM
310     PIVOT(I) = A(ICOLUM,ICOLUM)
320     DETERM = DETERM*PIVOT(I)
C
C     DIVIDE PIVOT BY PIVOT ELEMENT
C
330     A(ICOLUM,ICOLUM) = 1.0
340     DO 350 L=1,N
350     A(ICOLUM,L) = A(ICOLUM,L)/PIVOT(I)
355     IF (M .LE. 0) GOTO 380
360     DO 370 L=1,M
370     B(ICOLUM,L) = B(ICOLUM,L)/PIVOT(I)
C
C     REDUCE NON-PIVOT ROWS
C
380     DO 550 L1=1,N
390     IF (L1 .EQ. ICOLUM) GOTO 550
400     T = A(L1,ICOLUM)
420     A(L1,ICOLUM) = 0.0
430     DO 450 L=1,N
450     A(L1,L) = A(L1,L) - A(ICOLUM,L)*T
455     IF (M .LE. 0) GOTO 550
460     DO 500 L=1,M
500     B(L1,L) = B(L1,L) - B(ICOLUM,L)*T
550     CONTINUE
C
C     INTERCHANGE COLUMNS
C
600     DO 710 I=1,N
610     L = N + 1 - I
620     IF (INDEX(L,1) .EQ. INDEX(L,2)) GOTO 710
630     JROW = INDEX(L,1)
640     JCOLUM = INDEX(L,2)
650     DO 705 K=1,N
660     SWAP = A(K,JROW)
670     A(K,JROW) = A(K,JCOLUM)
700     A(K,JCOLUM) = SWAP
705     CONTINUE
710     CONTINUE
740     RETURN
      END
C-----

```

```

SUBROUTINE TRANS1(A1,A2,T)
C
C GET THE STRAIN TRANSFORMATION MATRIX (T)
C

```

```

DIMENSION T(6,6)
S1=SIND(A1)
S2=SIND(A2)
C1=COSD(A1)
C2=COSD(A2)
S1S=S1*S1
S2S=S2*S2
C1S=C1*C1
C2S=C2*C2
SC1=S1*C1
SC2=S2*C2
T(1,1)=C1S*C2S
T(2,1)=S1S*C2S
T(3,1)=S2S
T(4,1)=S1*SC2*2.0
T(5,1)=C1*SC2*2.0
T(6,1)=SC1*C2S*2.0
T(1,2)=S1S
T(2,2)=C1S
T(3,2)=0.0
T(4,2)=0.0
T(5,2)=0.0
T(6,2)=-SC1*2.0
T(1,3)=C1S*S2S
T(2,3)=S1S*S2S
T(3,3)=C2S
T(4,3)=-S1*SC2*2.0
T(5,3)=-C1*SC2*2.0
T(6,3)=SC1*S2S*2.0
T(1,4)=SC1*S2
T(2,4)=-T(1,4)
T(3,4)=0.0
T(4,4)=C1*C2
T(5,4)=-S1*C2
T(6,4)=-(C1S-S1S)*S2
T(1,5)=-C1S*SC2
T(2,5)=-S1S*SC2
T(3,5)=SC2
T(4,5)=S1*(C2S-S2S)
T(5,5)=C1*(C2S-S2S)
T(6,5)=-2.0*SC1*SC2
T(1,6)=-SC1*C2
T(2,6)=SC1*C2
T(3,6)=0.0
T(4,6)=C1*S2
T(5,6)=-S1*S2
T(6,6)=(C1S-S1S)*C2
RETURN
END

```

```

C-----
SUBROUTINE TRANS2(A1,A2,T)
C
C GET THE INVERSE STRAIN TRANSFORMATION MATRIX (T)
C

```

```

DIMENSION T(6,6)
S1=SIND(A1)
S2=SIND(A2)
C1=COSD(A1)
C2=COSD(A2)
S1S=S1*S1
S2S=S2*S2
C1S=C1*C1

```

```

C2S=C2*C2
SC1=S1*C1
SC2=S2*C2
T(1,1)=C1S*C2S
T(1,2)=S1S*C2S
T(1,3)=S2S
T(1,4)=S1*SC2
T(1,5)=C1*SC2
T(1,6)=SC1*C2S
T(2,1)=S1S
T(2,2)=C1S
T(2,3)=0.0
T(2,4)=0.0
T(2,5)=0.0
T(2,6)=-SC1
T(3,1)=C1S*S2S
T(3,2)=S1S*S2S
T(3,3)=C2S
T(3,4)=-S1*SC2
T(3,5)=-C1*SC2
T(3,6)=SC1*S2S
T(4,1)=2.0*SC1*S2
T(4,2)=-T(4,1)
T(4,3)=0.0
T(4,4)=C1*C2
T(4,5)=-S1*C2
T(4,6)=- (C1S-S1S)*S2
T(5,1)=-2.0*C1S*SC2
T(5,2)=-2.0*S1S*SC2
T(5,3)=2.0*SC2
T(5,4)=S1*(C2S-S2S)
T(5,5)=C1*(C2S-S2S)
T(5,6)=-2.0*SC1*SC2
T(6,1)=-2.0*SC1*C2
T(6,2)=2.0*SC1*C2
T(6,3)=0.0
T(6,4)=C1*S2
T(6,5)=-S1*S2
T(6,6)= (C1S-S1S)*C2
RETURN
END

```

C-----
SUBROUTINE GETSS(MM,MN,PROP,SS)

```

C
  DIMENSION SS(6,6),PROP(MM,10)
  DO 1113 II=1,6
  DO 1113 JJ=1,6
1113 SS(II,JJ)=0.0
  E1=PROP(MN,1)
  E2=PROP(MN,2)
  U1=PROP(MN,3)
  U2=PROP(MN,4)
  G1=PROP(MN,5)
  G2=PROP(MN,6)
  R=U1*U1*E2/E1
  D=(1.0+U2)*(1.0-U2-2.0*R)
  SS(1,1)=E1*(1.0-U2*U2)/D
  SS(1,2)=E2*U1*(1.0+U2)/D
  SS(1,3)=SS(1,2)
  SS(2,1)=SS(1,2)
  SS(2,2)=E2*(1.0-R)/D
  SS(2,3)=E2*(U2+R)/D
  SS(3,1)=SS(1,3)
  SS(3,2)=SS(2,3)
  SS(3,3)=SS(2,2)
  SS(4,4)=G2

```

```

SS(5,5)-G1
SS(6,6)-G1
RETURN
END

```

C-----
SUBROUTINE GETB(AA,BB,CC,X,Y,Z,BM)

```

C
  DIMENSION BM(6,24)
  DO 100 I=1,6
  DO 100 J=1,24
100 BM(I,J)=0.0
  X1--(0.125/AA)*(1.0-Y/BB)*(1.0-Z/CC)
  Y1--(0.125/BB)*(1.0-X/AA)*(1.0-Z/CC)
  Z1--(0.125/CC)*(1.0-X/AA)*(1.0-Y/BB)
  X2--(0.125/AA)*(1.0-Y/BB)*(1.0+Z/CC)
  Y2--(0.125/BB)*(1.0-X/AA)*(1.0+Z/CC)
  Z2--(0.125/CC)*(1.0-X/AA)*(1.0-Y/BB)
  X3--(0.125/AA)*(1.0+Y/BB)*(1.0-Z/CC)
  Y3--(0.125/BB)*(1.0-X/AA)*(1.0-Z/CC)
  Z3--(0.125/CC)*(1.0-X/AA)*(1.0+Y/BB)
  X4--(0.125/AA)*(1.0+Y/BB)*(1.0+Z/CC)
  Y4--(0.125/BB)*(1.0-X/AA)*(1.0+Z/CC)
  Z4--(0.125/CC)*(1.0-X/AA)*(1.0+Y/BB)
  X5--(0.125/AA)*(1.0-Y/BB)*(1.0-Z/CC)
  Y5--(0.125/BB)*(1.0+X/AA)*(1.0-Z/CC)
  Z5--(0.125/CC)*(1.0+X/AA)*(1.0-Y/BB)
  X6--(0.125/AA)*(1.0-Y/BB)*(1.0+Z/CC)
  Y6--(0.125/BB)*(1.0+X/AA)*(1.0+Z/CC)
  Z6--(0.125/CC)*(1.0+X/AA)*(1.0-Y/BB)
  X7--(0.125/AA)*(1.0+Y/BB)*(1.0-Z/CC)
  Y7--(0.125/BB)*(1.0+X/AA)*(1.0-Z/CC)
  Z7--(0.125/CC)*(1.0+X/AA)*(1.0+Y/BB)
  X8--(0.125/AA)*(1.0+Y/BB)*(1.0+Z/CC)
  Y8--(0.125/BB)*(1.0+X/AA)*(1.0+Z/CC)
  Z8--(0.125/CC)*(1.0+X/AA)*(1.0+Y/BB)
  BM(1,1)=X1
  BM(2,2)=Y1
  BM(3,3)=Z1
  BM(1,4)=X2
  BM(2,5)=Y2
  BM(3,6)=Z2
  BM(1,7)=X3
  BM(2,8)=Y3
  BM(3,9)=Z3
  BM(1,10)=X4
  BM(2,11)=Y4
  BM(3,12)=Z4
  BM(1,13)=X5
  BM(2,14)=Y5
  BM(3,15)=Z5
  BM(1,16)=X6
  BM(2,17)=Y6
  BM(3,18)=Z6
  BM(1,19)=X7
  BM(2,20)=Y7
  BM(3,21)=Z7
  BM(1,22)=X8
  BM(2,23)=Y8
  BM(3,24)=Z8
  BM(4,1)=Y1
  BM(5,2)=Z1
  BM(6,3)=X1
  BM(4,4)=Y2
  BM(5,5)=Z2
  BM(6,6)=X2
  BM(4,7)=Y3

```



```

BM(5,8)-Z3
BM(6,9)-X3
BM(4,10)-Y4
BM(5,11)-Z4
BM(6,12)-X4
BM(4,13)-Y5
BM(5,14)-Z5
BM(6,15)-X5
BM(4,16)-Y6
BM(5,17)-Z6
BM(6,18)-X6
BM(4,19)-Y7
BM(5,20)-Z7
BM(6,21)-X7
BM(4,22)-Y8
BM(5,23)-Z8
BM(6,24)-X8
BM(6,1)-Z1
BM(4,2)-X1
BM(5,3)-Y1
BM(6,4)-Z2
BM(4,5)-X2
BM(5,6)-Y2
BM(6,7)-Z3
BM(4,8)-X3
BM(5,9)-Y3
BM(6,10)-Z4
BM(4,11)-X4
BM(5,12)-Y4
BM(6,13)-Z5
BM(4,14)-X5
BM(5,15)-Y5
BM(6,16)-Z6
BM(4,17)-X6
BM(5,18)-Y6
BM(6,19)-Z7
BM(4,20)-X7
BM(5,21)-Y7
BM(6,22)-Z8
BM(4,23)-X8
BM(5,24)-Y8
RETURN
END

```

C-----

SUBROUTINE GETMS(MM,MN,PROP,S,SAFE)

C

```

DIMENSION S(6),PROP(MM,10)
SL=S(1)
IF(SL.GE.0.0) SLL=PROP(MN,7)
IF(SL.LT.0.0) SLL=PROP(MN,8)
SL=ABS(SL)
SLL=ABS(SLL)
IF(SLL.EQ.0.0) SLL=1.0
SAFE=(SLL-SL)/SLL
D=(S(2)+S(3))/2.0
B=(S(2)-S(3))/2.0
B=ABS(B)
H=S(4)/2.0
H=ABS(H)
R=B*B+H*H
R=SQRT(R)
MAX=D+R
MIN=R-D
IF(MAX.LE.0.0) GO TO 100
SLL=PROP(MN,9)
SLL=ABS(SLL)

```

```

      IF(SLL.EQ.0.0) SLL=1.0
      SAF=(SLL-MAX)/SLL
      IF(SAF.LT.SAFE) SAFE=SAF
100  CONTINUE
      IF(MIN.GE.0.0) GO TO 200
      SLL=PROP(MN,10)
      SLL=ABS(SLL)
      IF(SLL.EQ.0.0) SLL=1.0
      SAF=(SLL+MIN)/SLL
      IF(SAF.LT.SAFE) SAFE=SAF
200  CONTINUE
      RETURN
      END

```

C-----

SUBROUTINE GETDD(SSS1,SSS2,V1,V2,DD1,DD2)

C

```

      DIMENSION SSS1(6,6),SSS2(6,6),DD1(6,6),DD2(6,6)
      DIMENSION DD(6,6),CB(6,6)
      DO 7700 II=1,6
      DO 7700 JJ=1,6
      CB(II,JJ)=SSS2(II,JJ)-SSS1(II,JJ)
      DD(II,JJ)=0.0
      DD1(II,JJ)=0.0
7700  DD2(II,JJ)=0.0
      DD(1,1)=SSS1(1,1)
      DD(1,2)=SSS2(1,1)
      DD(1,3)=SSS1(1,5)
      DD(1,4)=SSS2(1,5)
      DD(1,5)=SSS1(1,6)
      DD(1,6)=SSS2(1,6)
      DD(2,1)=V1
      DD(2,2)=V2
      DD(3,1)=SSS1(5,1)
      DD(3,2)=SSS2(5,1)
      DD(3,3)=SSS1(5,5)
      DD(3,4)=SSS2(5,5)
      DD(3,5)=SSS1(5,6)
      DD(3,6)=SSS2(5,6)
      DD(4,3)=V1
      DD(4,4)=V2
      DD(5,1)=SSS1(6,1)
      DD(5,2)=SSS2(6,1)
      DD(5,3)=SSS1(6,5)
      DD(5,4)=SSS2(6,5)
      DD(5,5)=SSS1(6,6)
      DD(5,6)=SSS2(6,6)
      DD(6,5)=V1
      DD(6,6)=V2
      CALL INV(DD)
      DD1(1,1)=DD(1,2)
      DD1(1,5)=DD(1,4)
      DD1(1,6)=DD(1,6)
      DD1(1,2)=DD(1,1)*CB(1,2)+DD(1,3)*CB(5,2)+DD(1,5)*CB(6,2)
      DD1(1,3)=DD(1,1)*CB(1,3)+DD(1,3)*CB(5,3)+DD(1,5)*CB(6,3)
      DD1(1,4)=DD(1,1)*CB(1,4)+DD(1,3)*CB(5,4)+DD(1,5)*CB(6,4)
      DD1(2,2)=1.0
      DD1(3,3)=1.0
      DD1(4,4)=1.0
      DD1(5,1)=DD(3,2)
      DD1(5,5)=DD(3,4)
      DD1(5,6)=DD(3,6)
      DD1(5,2)=DD(3,1)*CB(1,2)+DD(3,3)*CB(5,2)+DD(3,5)*CB(6,2)
      DD1(5,3)=DD(3,1)*CB(1,3)+DD(3,3)*CB(5,3)+DD(3,5)*CB(6,3)
      DD1(5,4)=DD(3,1)*CB(1,4)+DD(3,3)*CB(5,4)+DD(3,5)*CB(6,4)
      DD1(6,1)=DD(5,2)
      DD1(6,5)=DD(5,4)

```

```

DD1(6,6)=DD(5,6)
DD1(6,2)=DD(5,1)*CB(1,2)+DD(5,3)*CB(5,2)+DD(5,5)*CB(6,2)
DD1(6,3)=DD(5,1)*CB(1,3)+DD(5,3)*CB(5,3)+DD(5,5)*CB(6,3)
DD1(6,4)=DD(5,1)*CB(1,4)+DD(5,3)*CB(5,4)+DD(5,5)*CB(6,4)
DD2(1,1)=DD(2,2)
DD2(1,5)=DD(2,4)
DD2(1,6)=DD(2,6)
DD2(1,2)=DD(2,1)*CB(1,2)+DD(2,3)*CB(5,2)+DD(2,5)*CB(6,2)
DD2(1,3)=DD(2,1)*CB(1,3)+DD(2,3)*CB(5,3)+DD(2,5)*CB(6,3)
DD2(1,4)=DD(2,1)*CB(1,4)+DD(2,3)*CB(5,4)+DD(2,5)*CB(6,4)
DD2(2,2)=1.0
DD2(3,3)=1.0
DD2(4,4)=1.0
DD2(5,1)=DD(4,2)
DD2(5,5)=DD(4,4)
DD2(5,6)=DD(4,6)
DD2(5,2)=DD(4,1)*CB(1,2)+DD(4,3)*CB(5,2)+DD(4,5)*CB(6,2)
DD2(5,3)=DD(4,1)*CB(1,3)+DD(4,3)*CB(5,3)+DD(4,5)*CB(6,3)
DD2(5,4)=DD(4,1)*CB(1,4)+DD(4,3)*CB(5,4)+DD(4,5)*CB(6,4)
DD2(6,1)=DD(6,2)
DD2(6,5)=DD(6,4)
DD2(6,6)=DD(6,6)
DD2(6,2)=DD(6,1)*CB(1,2)+DD(6,3)*CB(5,2)+DD(6,5)*CB(6,2)
DD2(6,3)=DD(6,1)*CB(1,3)+DD(6,3)*CB(5,3)+DD(6,5)*CB(6,3)
DD2(6,4)=DD(6,1)*CB(1,4)+DD(6,3)*CB(5,4)+DD(6,5)*CB(6,4)
RETURN
END

```

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE February 1993	3. REPORT TYPE AND DATES COVERED Contractor Report	
4. TITLE AND SUBTITLE Approximating the Stress Field Within the Unit Cell of a Fabric Reinforced Composite Using Replacement Elements			5. FUNDING NUMBERS C NAS1-19000 WU 505-63-50-04	
6. AUTHOR(S) R. L. Foye				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Lockheed Engineering and Sciences Company Research Drive Hampton, VA 23666			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Langley Research Center Hampton, VA 23681			10. SPONSORING / MONITORING AGENCY REPORT NUMBER NASA CR-191422	
11. SUPPLEMENTARY NOTES Langley Technical Monitor: Charles E. Harris Final Report				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Unclassified - Unlimited Subject Category 24			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) This report concerns the prediction of the elastic moduli and the internal stresses within the unit cell of a fabric reinforced composite. In the proposed analysis no restrictions or assumptions are necessary concerning yarn or tow cross-sectional shapes or paths through the unit cell but the unit cell itself must be a right hexagonal parallelepiped. All the unit cell dimensions are assumed to be small with respect to the thickness of the composite structure that it models. The finite element analysis of a unit cell is usually complicated by the mesh generation problems and the non-standard, adjacent-cell, boundary conditions. This analysis avoids these problems through the use of preprogrammed boundary conditions and replacement materials (or elements). With replacement elements it is not necessary to match all the constitutional material interfaces with finite element boundaries. Simple brick-shaped elements can be used to model the unit cell structure. The analysis predicts the elastic constants and the average stresses within each constituent material of each brick element. The application and results of this analysis are demonstrated through several example problems which include a number of composite microstructures.				
14. SUBJECT TERMS Textile composites; Stress analysis; Unit cell; Replacement elements; Finite-elements			15. NUMBER OF PAGES 138	
			16. PRICE CODE A07	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT	

